The ADER path to high-order schemes for hyperbolic equations

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Sixty years ago, Godunov introduced his method to solve the Euler equations of gas dynamics, thus creating the Godunov school of thought for numerical approximation of hyperbolic conservation laws. The building block of the original first-order Godunov upwind method is the conventional piece-wise constant data Riemann problem.

ADER is an approach to construct high-order extensions of the Godunov first-order method, in which the conventional Riemann problem is replaced by the generalized Riemann problem (GRPk), a piece-wise smooth data Cauchy problem that may include stiff source terms. The resulting schemes are of arbitrary k+1 order accuracy in both space and time; there is no theoretical accuracy barrier. There are by now several methods for solving the generalized Riemann problem, thus giving rise to several classes of ADER schemes.

ADER is a fully discrete methodology that operates in the finite volume and DG finite elements frameworks. These schemes are orders-of-magnitude cheaper than first-order methods for attaining a prescribed, small error and are therefore mandatory in ambitious scientific and technological applications. Here I review some key aspects of the ADER methodology and discuss its strengths, shortcomings and issues of current research interest. Sample applications will be shown.

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