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The Cartesian Grid Active Flux Method with Adaptive Mesh Refinement

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The Active Flux method is a third order accurate finite volume method for hyperbolic conservation laws, that was previously introduced by Eymann, Roe and coauthors [2, 3, 4, 5, 6], which uses a continuous, piecewise quadratic reconstruction and Simpson's rule to compute numerical fluxes. While classical finite volume methods only use cell average values of the conserved quantities as degrees of freedom, the Active Flux method uses also point values at grid cell interfaces at the current time as well as at later time levels. These point values together with the cell average value are also used to compute the reconstruction:

We consider two-dimensional hyperbolic conservation laws in divergence form

$$\partial_t q + \partial_x f(q) + \partial_y g(q) = 0,$$

where $q : \mathbb{R}^2 \times \mathbb{R}^+ \to \mathbb{R}^s$ is a vector of conserved quantities and $f, g : \mathbb{R}^s \to \mathbb{R}^s$ are vector valued flux functions. The grid cell (i, j) is described by $\left[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}\right] \times \left[y_{j-\frac{1}{2}}, y_{j+\frac{1}{2}}\right] \subset \mathbb{R}^2$, $i, j, \in \mathbb{Z}$. We compute cell averaged values of the conserved quantities with a classical finite volume method update of the form

$$Q_{ij}^{n+1} = Q_{ij}^n - \frac{\Delta t}{\Delta x}\left(F_{i+\frac{1}{2},j} - F_{i-\frac{1}{2},j}\right) - \frac{\Delta t}{\Delta y}\left(G_{i,j+\frac{1}{2}} - G_{i,j-\frac{1}{2}}\right),$$

where $\Delta x = x_{i-\frac{1}{2}} - x_{i+\frac{1}{2}}$ and $\Delta y = y_{i-\frac{1}{2}} - y_{i+\frac{1}{2}}$. $F_{i\pm\frac{1}{2},j}$, $G_{i,j\pm\frac{1}{2}}$ are numerical fluxes at the grid cell interfaces, i.e.

$$F_{i+\frac{1}{2},j} \approx \frac{1}{\Delta t \Delta y} \int_{t_n}^{t_{n+1}} \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} f(q(x_{i+\frac{1}{2}}, y, t)) dy\, dt$$

which are computed using Simpson's rule

$$\begin{aligned} F_{i+\frac{1}{2},j} := \frac{1}{36}\Big(& f(Q^n_{i+\frac{1}{2},j-\frac{1}{2}}) + 4f(Q^n_{i+\frac{1}{2},j}) + f(Q^n_{i+\frac{1}{2},j+\frac{1}{2}}) \\ & + 4f(Q^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2}}) + 16f(Q^{n+\frac{1}{2}}_{i+\frac{1}{2},j}) + 4f(Q^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2}}) \\ & + f(Q^{n+1}_{i+\frac{1}{2},j-\frac{1}{2}}) + 4f(Q^{n+1}_{i+\frac{1}{2},j}) + f(Q^{n+1}_{i+\frac{1}{2},j+\frac{1}{2}})\Big) \end{aligned} \tag{1}$$

and analogously for the remaining fluxes.

The Q values, which appear at the right hand side of (1), are approximations of point values of the conserved quantities along the grid cell interface at different times, which can be computed using e.g. an exact evolution formula for linear advection and acoustics applied to the piecewise quadratic reconstrucion.

We implemented the adaptive Active Flux method as a new solver in ForestClaw [1], a software for parallel adaptive mesh refinement based on a quadtree approach, developed by Calhoun and Burstedde based on the p4est software.

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Mesh refinement is realised by a bisection of grid patches, where single grid patches can efficiently be handled by separate processors in a parallel computation. Figure 1 shows a typical situation applying the Active Flux method to an advection equation with space-dependent velocity field with refinement level $2-4$. The local stencil of the Active Flux method also allows an efficient implementation of subcycling, meaning that the global time is not restricted by the finest patch, but several time steps are performed on refined patches, while one time step is performed on the coarsest grid.

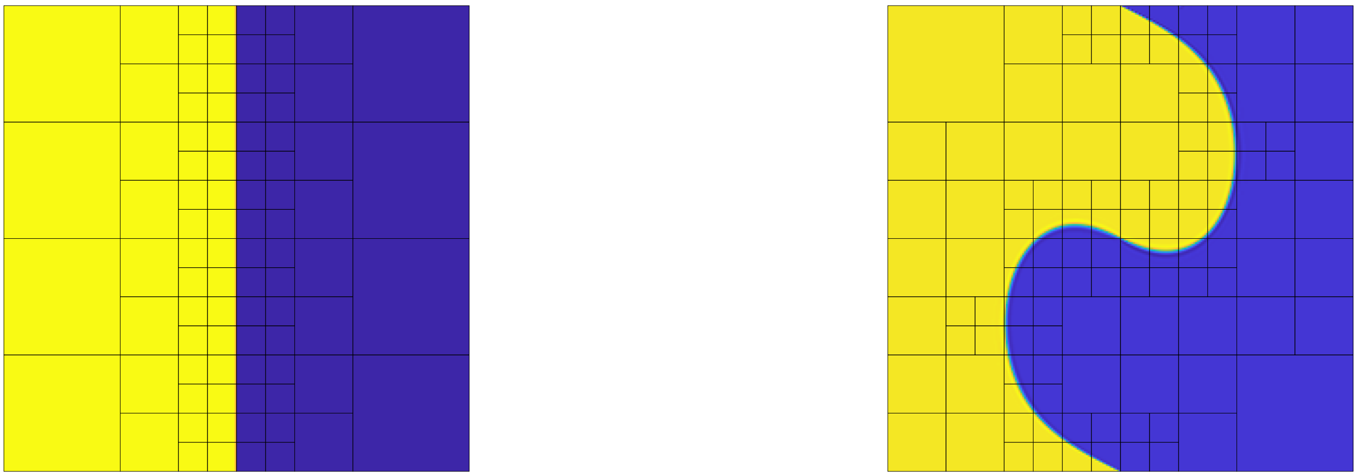

Figure 1: Advection with space-dependent velocity field and Riemann initial condition for refinement level $2-4$.

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