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Some Existence Results for Non-Local Scalar Conservation Laws with Discontinuous Flux

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We are concerned with the following class of space-discontinuous scalar conservation laws given by (1) with a non-local flux arising in traffic modeling:

$$\rho_t + \Big(W[\rho,\eta,a,b](t)F(t,x,\rho)\Big)_x \;=\; 0\;, \tag{1}$$

where

$$F(t,x,\rho) = \left(H(-x)g(\rho) + H(x)f(\rho)\right),$$

ρ is the the unknown density, H the Heavyside function and W is a nonlocal flux of the type

$$W[\rho,\eta,a,b](t) = \int\limits_{a(t)}^{b(t)} \eta(s,t)\rho(s,t)ds. \tag{2}$$

W is called "non-local impact" and essentially scales the flux function F by incorporating the effects of the density of traffic/crowd between the spatial points $a(t)$ and $b(t)$. This could happen due to the presence of tolls/traffic jams or columns present in constrained room settings. The non–local term is realized via a spatial integration of the solution ρ between specified boundaries and affects the flux function of a given "local" conservation law in a multiplicative way, on either side of the interface $x = 0$. η represents a weight of this nonlocal impact W in space and time, and finally a and b represent the "boundaries" of that nonlocal term in space. Also, $g, f : \mathbb{R} \times \mathbb{R}^+$ are sufficiently smooth functions and can be seen as a model to depict the flow of the car/crowd on left and right side of the junction. Depending on the nature of f, g and W, these equations are known to model various physical phenomena such as crowd dynamics, traffic flow, biological processes, semiconductor operations, etc. The *nonlocal* nature of (1) is particularly suitable in describing the behavior of crowds, where each member moves according to her/his evaluation of the crowd density and its variations within her/his horizon and several models have been recently considered in the literature, e.g. [1, 2, 3, 4, 5, 6], where in particular, the velocity vector field was assumed to be sufficiently smooth. This model with $f = g$ was recently studied in [7], where existence and uniqueness were proven using fixed point arguments.

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The aim is to extend the results for $f \neq g$ and establish the existence and uniqueness of weak entropy solutions via the convergence of numerical schemes. The result will generalize the existing results both in the literature of nonlocal conservation laws and also scalar conservation laws for discontinuous flux. Our approach is based on the convergence of a Lax-Friedrichs type algorithm that yields a sequence of approximate solutions to (1) that, up to a subsequence, converges to a weak entropy solution of (1). The proof is based on obtaining a uniform bound on the total variation of the approximate solutions.

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