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A multiclass Lighthill-Whitham-Richards traffic model with a discontinuous velocity function

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The well-known Lighthill-Whitham-Richards (LWR) kinematic model of traffic flow models the evolution of the local density of cars by a nonlinear scalar conservation law. The transition between free and congested flow regimes can be described by a flux or velocity function that has a discontinuity at a determined density. A numerical scheme to handle the resulting LWR model with discontinuous flux function and Dirichlet boundary condition was proposed in [1]. In this talk, we present a similar scheme proposed in [2], which was constructed by decomposing the discontinuous velocity function into a Lipschitz continuous function plus a Heaviside function and designing a corresponding splitting scheme. The part of the scheme related to the discontinuous flux is handled by a semi-implicit step that does, however, not involve the solution of systems of linear or nonlinear equations. It is proved that the whole scheme converges to a weak solution in the scalar case. The scheme can in a straightforward manner be extended to the multiclass LWR (MCLWR) model, which is defined by a hyperbolic system of N conservation laws for N driver classes that are distinguished by their preferential velocities. It is shown that the multiclass scheme satisfies an invariant region principle, that is, all densities are nonnegative and their sum does not exceed a maximum value. In the scalar and multiclass cases no flux regularization or Riemann solver is involved, and the CFL condition is not more restrictive than for an explicit scheme for the continuous part of the flux. Numerical tests for the scalar and multiclass cases are presented with Dirichlet and periodic boundary conditions.

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