XVIII INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS
THEORY, NUMERICS, APPLICATIONS
Málaga, June 20-24, 2022

On thermodynamically compatible schemes for continuum mechanics

Saray BUSTO *, Michael DUMBSER †

In this talk we present two new families of semi-discrete and fully-discrete finite volume (FV) and discontinuous Galerkin (DG) finite element schemes for overdetermined, hyperbolic and thermodynamically compatible PDE systems. In the following we will denote these methods as HTC schemes. In particular, we consider the Euler equations of compressible gasdynamics, as well as the more complex Godunov-Peshkov-Romenski (GPR) model of continuum mechanics, which, at the aid of suitable relaxation source terms, is able to describe nonlinear elasto-plastic solids at large deformations as well as viscous fluids as two special cases of a more general first order hyperbolic model of continuum mechanics, see [3, 2, 4]. The main novelty of the schemes presented in this paper lies in the fact that we solve the *entropy inequality* as a primary evolution equation rather than the usual total energy conservation law. Instead, total energy conservation is achieved as a mere consequence of a thermodynamically compatible discretization of all the other equations. For this, we first construct a discrete framework for the compressible Euler equations that mimics the continuous framework of Godunov's seminal paper *An interesting class of quasilinear systems* of 1961, see [1], *exactly* at the discrete level. All other terms in the governing equations of the more general GPR model, including non-conservative products, are judiciously discretized in order to achieve discrete thermodynamic compatibility, with the exact conservation of total energy density as a direct consequence of all the other equations, see [5, 6]. As a result, the HTC schemes proposed in this talk are provably marginally stable in the energy norm and satisfy a discrete entropy inequality by construction. We show some computational results obtained with HTC FV and DG schemes in one and two space dimensions, considering both the fluid limit as well as the solid limit of the governing partial differential equations.

Acknowledgements

S.B. and M.D. are members of the INdAM GNCS group and acknowledge the financial support received from the Italian Ministry of Education, University and Research (MIUR) via the PRIN 2017 project *Innovative numerical methods for evolutionary partial differential equations and applications*. S.B. was also funded by INdAM via a GNCS grant for young researchers and by an *UniTN starting grant* of the University of Trento.

References

[1] S.K. Godunov. An interesting class of quasilinear systems. *Dokl. Akad. Nauk SSSR*, 139(3):521–523, 1961.

[2] S.K. Godunov and E.I. Romenski. *Elements of continuum mechanics and conservation laws*. Kluwer Academic/Plenum Publishers, 2003.

[3] E.I. Romenski. Hyperbolic systems of thermodynamically compatible conservation laws in continuum mechanics. *Math. Comput. Modell.*, 28(10):115–130, 1998.

[4] M. Dumbser, I. Peshkov, E. Romenski, and O. Zanotti. High order ADER schemes for a unified first order hyperbolic formulation of continuum mechanics: Viscous heat–conducting fluids and elastic solids. *J. Comput. Phys.*, 314:824–862, 2016.

[5] S. Busto, M. Dumbser, S. Gavrilyuk, and K. Ivanova. On thermodynamically compatible finite volume methods and path-conservative ADER discontinuous Galerkin schemes for turbulent shallow water flows. *Journal of Scientific Computing*, 88:28, 2021.

[6] S. Busto, M. Dumbser, I. Peshkov and E. Romenski. On thermodynamically compatible finite volume schemes for continuum mechanics. *SIAM Journal on Scientific Computing*, to appear.

*University of Trento, Via Mesiano 77, I-38123 Trento, Italy. Email: saray.busto@unitn.it
†University of Trento, Via Mesiano 77, I-38123 Trento, Italy. Email: michael.dumbser@unitn.it