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A class of conservative L^2-stable schemes for the compressible Euler equations on staggered grids

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Finite volume schemes on staggered grids are popular among thermalhydraulics engineers for their good low Mach number asymptotic expansion and the absence of checkerboard type spurious oscillations [5]. However, they are generally non conservative, and their stability analysis has historically been performed with a heuristic approach and the tuning of numerical parameters ([4]).

In this talk, we first investigate the L^2-stability of staggered schemes by analysing their numerical diffusion operator. This analysis of the numerical diffusion operator gives new insights into the schemes numerical properties and is a step towards a proof of linear stability or stability for almost constant initial data. For most classical staggered schemes ([5, 1, 2, 3]), our analysis shows that the numerical diffusion is highly nonlinear and we are able to prove its positivity only in the case of constant sign velocities. In order to deal with compressible flows, possibly at high Mach number, we then propose a new class of conservative linearly L^2-stable staggered schemes for the compressible Euler equations on staggered grids. The schemes are based on a carefully chosen numerical diffusion operator and the proof of stability follows from the energy decay in the basis that symetrises the Euler equations. An important remark is that unlike Godunov type schemes on colocated grids, the numerical diffusion operator of a a symmetric system is not symmetric. This property is important to avoid spurious checkerboard modes oscillations.

We give an example of such a conservative staggered scheme and present some numerical results showing the good behaviour of the method for low and high Mach number flows, in 1D and 2D.

References

[1] R. Herbin, W. Kheriji, and J.-C. Latché. Staggered schemes for all speed flows. *ESAIM: Proceedings, EDP Sciences, Congrès National de Mathématiques Appliquées et Industrielles, 35, pp.122-150*, 2011.

[2] R. Herbin, W. Kheriji, and J.-C. Latché. On some implicit and semi-implicit staggered schemes for the shallow water and euler equations. *ESAIM: Mathematical Modelling and Numerical Analysis, EDP Sciences, 48 (6), pp.1807-1857*, 2014.

[3] R. Herbin and J.-C. Latché. A kinetic energy preserving convection operator for the mac discretization of compressible navier-stokes equations. *Mathematical Modelling and Numerical Analysis*, 2010. `https://hal.archives-ouvertes.fr/hal-00477079/document`.

[4] C. W. Hirt. Heuristic stability theory for finite difference equations. *J. Comp. Phys., 2, pp. 339-355*, 1968.

[5] A. Prosperetti and G. Tryggvason. *Computational methods for multiphase flow*. Cambridge University press, 2009.

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