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New smoothness indicators for Hamilton-Jacobi equations applied to Image Segmentation

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A common and crucial feature in the approximation of weak solutions of nonlinear hyperbolic equations is the lack of smoothness. This has motivated several efforts to define appropriate indicators, based on the values of the approximate solutions, in order to detect the singularity regions of the domain. This information helps to adapt the approximation scheme in order to avoid spurious oscillations when using high-order schemes. In [2] we propose a genuinely multidimensional extension of the WENO procedure in order to overcome the limitations of indicators based on dimensional splitting. Our aim is to obtain new regularity indicators for problems in 2D and apply them to a class of "adaptive filtered"schemes for first order evolutionary Hamilton-Jacobi equations, which has been proven to be convergent in [1]. These filtered schemes are obtained by coupling a high-order (possibly unstable) scheme and a monotone one. The mixture is governed by a filter function and by a positive switching parameter which goes to zero as the time and space steps are going to 0. The adaptivity is related to the smoothness indicators and allows to automatically tune the switching parameter in time and space. We want to apply this kind of filtered schemes to the image segmentation problem [3], whose purpose is to detect the boundaries of the object(s) represented inside a given picture. To do that, we follow the Level-Set (LS) approach, introduced by Osher and Sethian in the 1980s [4, 6] and then used to deal with several applications, e.g., front propagation, computer vision, and computational fluid dynamics (see the monographs by Sethian [7] and by Osher and Fedkiw [5] for several interesting examples). This method is nowadays very popular due to its simplicity and its capability to deal with topological changes. In fact, the main advantages of the LS method are the possibility to easily describe time-varying objects, follow shapes that change topology, for example when a shape splits in two, develop holes, or to do the reverse of these operations. For the image segmentation problem, the application of the LS method is based on the evolution of a curve according to a normal velocity based on the gray levels of the image; typically, the curve is described as the 0-level set of a representation function (or LS function). In $\mathbb{R}^2$, the LS method corresponds to defining an initial closed curve Γ_0 using an auxiliary function v_0 which has to change sign on Γ_0. The evolution of that curve at time t is denoted by Γ_t and is represented by the 0-level set of a function v, i.e.,

$$\Gamma_t := \{(x, y) : v(t, x, y) = 0\}.$$

This function v is the unique viscosity solution of the following evolutive nonlinear equation of Hamilton–Jacobi type:

$$\begin{cases} v_t(t,x) + c(x)|\nabla v(t,x)| = 0, & (t,x) \in [0,T] \times \mathbb{R}^2, \\ v(0,x) = v_0(x), & x \in \mathbb{R}^2, \end{cases}$$

where ∇v denotes the spatial gradient of v.

In this talk, we will show how solving the image segmentation problem using this new smoothness indicators applied to the adaptive filtered schemes, showing the effectiveness of the proposed indicators and the efficiency of the proposed scheme compared to other methods on numerical tests related to synthetic and real images.

Joint work with M. Falcone and G. Paolucci.

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