XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

A Non-Strictly Hyperbolic System for Compressible Two Phase Flows

Ferdinand Thein [*], Evgeniy Romenski [†], Michael Dumbser [‡]

In several previous works Godunov, Romenski and co-authors proposed a PDE model derived using fundamental principles, cf. [3, 4]. The governing PDE system belongs to the class of symmetric hyperbolic thermodynamically compatible systems (SHTC). Particular results on two fluid models were obtained by Romenski, Toro and co-authors [6, 5]. Numerical results for this type of model can exemplary be found in recent works by Dumbser, Romenski et al. e.g. [1, 2]. However, a distinguished analytical treatment is still far from being complete. As a first attempt we want to discuss the Riemann problem for the homogeneous barotropic (i.e. isentropic or isothermal) two fluid model derived from the SHTC system. The PDE system for compressible two-phase flows including (hyperbolic) heat conduction was previously discussed in Romenski et al. [6, 5]. The barotropic subsystem we want to discuss reads

$$\frac{\partial \alpha_1 \rho}{\partial t} + \frac{\partial \alpha_1 \rho u}{\partial x} = 0, \tag{1a}$$

$$\frac{\partial \alpha_1 \rho_1}{\partial t} + \frac{\partial \alpha_1 \rho_1 u_1}{\partial x} = 0, \tag{1b}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0, \tag{1c}$$

$$\frac{\partial(\alpha_1\rho_1 u_1 + \alpha_2 \rho_2 u_2)}{\partial t} + \frac{\partial\left(\alpha_1\rho_1 u_1^2 + \alpha_2\rho_2 u_2^2 + \alpha_1 p_1(\rho_1) + \alpha_2 p_2(\rho_2)\right)}{\partial x} = 0, \tag{1d}$$

$$\frac{\partial w}{\partial t} + \frac{\partial\left(\frac{1}{2}u_1^2 - \frac{1}{2}u_2^2 + \Psi_1(\rho_1) - \Psi_2(\rho_2)\right)}{\partial x} = 0, \tag{1e}$$

Here, α_1 is the volume fraction of the first phase which is connected with the volume fraction of the second phase α_2 by the saturation law $\alpha_1 + \alpha_2 = 1$, ρ is the mixture mass density which is connected with the phase mass densities ρ_1, ρ_2 by the relation $\rho = \alpha_1\rho_1 + \alpha_2\rho_2$. The phase mass fractions are defined as $c_1 = \alpha_1\rho_1/\rho$, $c_2 = \alpha_2\rho_2/\rho$ and it is easy to see that $c_1 + c_2 = 1$. Finally, $u = c_1 u_1 + c_2 u_2$ is the mixture velocity, $w = u_1 - u_2$ is the phase relative velocity. The equations describe the balance law for the volume fraction, the balance law for the mass fraction, the conservation of total mass, the total momentum conservation law, the balance for the relative velocity For the last equation we introduced

$$\Psi_i(\rho_i) = \begin{cases} h_i(\rho_i), \text{ isentropic} \\ g_i(\rho_i), \text{ isothermal} \end{cases}$$

where h_i is the specific enthalpy of the corresponding phase and g_i the specific Gibbs potential, respectively. We will present exact relations for the appearing waves, discuss the wave structure of the solution and further properties of the system under consideration. Due to the non-strictly hyperbolic character of the system we also present wave interactions barely discussed in the literature. Comparisons of numerical and exact solutions will be shown. Results presented here are also available in [7].

[*]RWTH Aachen, Templergraben 55, 52056 Aachen. Email: thein@igpm.rwth-aachen.de
[†]Sobolev Institute, Novosibirsk, Russia, Email: evrom@math.nsc.ru
[‡]University of Trento, Trento, Italy, Email: michael.dumbser@unitn.it

References

[1] M. Dumbser, I. Peshkov, E. Romenski, and O. Zanotti. High order ader schemes for a unified first order hyperbolic formulation of continuum mechanics: Viscous heat-conducting fluids and elastic solids. *Journal of Computational Physics*, 314:824 – 862, 2016.

[2] S. Busto, M. Dumbser, I. Pehskov, and E. Romenski. On thermodynamically compatible finite volume schemes for continuum mechanics. *SIAM Journal on Scientific Computing*, in press.

[3] S. K. Godunov. An interesting class of quasi-linear systems. *Dokl. Akad. Nauk SSSR*, 139:521–523, 1961.

[4] S. K. Godunov and E. I. Romenski. Nonstationary equations of nonlinear elasticity theory in eulerian coordinates. *Journal of Applied Mechanics and Technical Physics*, 13(6):868–884, Nov 1972.

[5] E. Romenski, D. Drikakis, and E. Toro. Conservative models and numerical methods for compressible two-phase flow. *Journal of Scientific Computing*, 42(1):68, Jul 2009.

[6] E. Romenski, A. D. Resnyansky, and E. F. Toro. Conservative hyperbolic formulation for compressible two-phase flow with different phase pressures and temperatures. *Quart. Appl. Math.*, 65(2):259–279, 2007.

[7] F. Thein, E. Romenski, M. Dumbser. Exact and numerical solutions of the Riemann problem for a conservative model of compressible two-phase flows. http://arxiv.org/abs/2203.12422