

# Well-balanced schemes for nearly steady adiabatic flow

Roger Käppeli \*

We present well-balanced finite volume schemes for the Euler equations with gravitation capable of preserving a discrete form of steady adiabatic flow. In many astrophysical applications, the dynamics of interest are realized near steady flow such as in accretion or wind phenomena. The schemes are not tied to any particular numerical flux and work for Cartesian geometry as well as for curvilinear coordinates such as cylindrical and spherical. In particular, the geometric source terms are discretized carefully to guarantee the well-balancedness of the schemes. Moreover, the schemes can handle arbitrary convex equations of state, which is especially important in astrophysical applications. We discuss second-order accurate schemes and their extension to multiple space dimensions. We will also discuss going beyond spatial second-order accuracy. The improved performance of the well-balanced schemes compared to standard schemes is demonstrated in several challenging numerical experiments. In particular, we will look at a stellar accretion phenomenon known as the standing accretion shock instability. The latter is believed to play an important role in the death of massive stars in a core-collapse supernova.

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\*Seminar for Applied Mathematics (SAM), Department of Mathematics, ETH Zurich, Rämistrasse 101 CH-8092 Zurich, Switzerland.  
roger.kaeppli@sam.math.ethz.ch