XVIII INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS
THEORY, NUMERICS, APPLICATIONS
Málaga, June 20-24, 2022

A multi-scale multi-lane model for heterogeneous traffic flows

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We consider a first order macroscopic multi-lane model [1, 2] of the form

$$
\begin{cases} \partial_t \rho_j + \partial_x F_j(\rho_j) = \dfrac{1}{\tau}\left(S_{j-1}(\rho_{j-1},\rho_j) - S_j(\rho_j,\rho_{j+1})\right), & x \in \mathbb{R},\ t \geq 0, \\ \rho_j(0,x) = \rho_j^0(x), & j = 1,\ldots,M. \end{cases} \tag{1}
$$

Above, M is the number of lanes on the road, $\rho_j : [0,+\infty[\times\mathbb{R} \to [0,R_j]$ is the vehicle density on lane j, $F_j(\rho_j) = \rho_j v_j(\rho_j)$ is the flux function and the average speed $v_j = v_j(\rho_j) : [0,R_j] \to [0,V_j]$ is a decreasing function such that $v_j(0) = V_j$ and $v_j(R_j) = 0$, so that F_j is concave. The source terms S_j account for mass exchanges form lane j to lane $j+1$ (setting $S_0 = S_M = 0$), scaled by a relaxation factor $1/\tau \in \mathbb{R}^+$.

To account for the presence of a moving bottleneck [3, 4] in lane j, we let $y : [0,+\infty[\to \mathbb{R}$ be its trajectory. The coupling with (1) is realized though the following microscopic ODE and constraint:

$$
\dot{y}(t) = \min\{u(t), v_j(\rho_j(t,y(t)+))\}, \qquad t > 0, \tag{2a}
$$

$$
y(0) = y_0, \tag{2b}
$$

$$
\rho_j(t,y(t))\left(v_j(\rho_j(t,y(t))) - \dot{y}(t)\right) \leq 0, \qquad t > 0, \tag{2c}
$$

where $u : [0,+\infty[\to [0,V_j]$ is the bottleneck's maximal speed.

We investigate the well-posedness of (1)-(2) and the relaxation limit as $\tau \to 0$.

References

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