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Exact Riemann solver for nonconvex special relativistic hydrodynamics

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The equations of special relativistic hydrodynamics (SRHD) form a nonlinear hyperbolic system of conservation laws which is closed with the constitutive relations represented by an equation of state (EoS) characterizing the equilibrium thermodynamic properties of the considered material. The thermodynamics, through the EoS, provides the classical or nonclassical (convex or nonconvex) character of the wave structure [3].

The convexity of the EoS is related to the curvature of the isentropes (lines at constant entropy) in the pressure-volume plane. For the SRHD system, the convexity of the system is given by the nonlinearity factor, which takes also into consideration the relativistic sound speed of the fluid [1].

The EoS is convex if the nonlinearity factor keeps it sign during the whole domain, and nonconvex if it has zeros. A convex EoS induces convex dynamics, where the possible waves are simple waves: expansive rarefactions and compressive shocks. Meanwhile, nonconvex dynamics allow for the formation of compound waves. These are structures of two or more simple waves travelling together.

The Riemann problem is a type of initial condition for the equations of hydrodynamics. It consists in two constant states separated by a jump discontinuity in one or more of the conserved magnitudes. When we let it develop, waves generate from the discontinuity and travel to each of the constant states. In convex dynamics the two waves can be either shocks or rarefactions, their nature determined by the initial conditions. In nonconvex dynamics combinations of simple waves arise at both sides of the contact discontinuity.

The numerical approximation of SRHD equations is still a challenge. Knowing the exact solution of Riemann problems has been a valuable resource at designing and validating numerical schemes, not only in the relativistic context but for the classical Euler equations.

In SRHD the exact solution of the Riemann problem has been studied for convex dynamics, using the ideal gas EoS [6]. The exact solution for nonconvex dynamics was studied in [7] for the Euler equations. Here, we present the exact solution for the Riemann problem in nonconvex SRHD. In particular, we use the phenomenological EoS presented in [2], that displays nonconvex regions with the appropriate value of parameters.

We design a strategy to solve the compound waves arising in the complex dynamics of nonconvex SHRD following the concept of mixed curves introduced by Liu [4]. We present the exact solution of the Riemann problems addressed in [5] including relativistic blast waves.

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