Adapted entropy solution for conservation laws with BV spatial flux

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We consider the initial value problem for a scalar conservation law in one space dimension with a spatially discontinuous flux. There may be infinitely many flux discontinuities, and the set of discontinuities may have accumulation points. Thus the existence of traces cannot be assumed. We present a Godunov type numerical scheme [2] which converges to the 'adapted entropy solution' [1] for nonlinear flux. We also consider fluxes having flat region and proved [4] the wellposedness of the entropy solution. In addition, a sufficient condition on the initial data and flux is coined to ensure a uniform BV bound on the entropy solutions. Counterexamples are constructed to exhibit the optimality of our assumptions. We further prove the convergence and rate of convergence of a Godunov type finite volume scheme to the entropy solution for a more general class of fluxes (Panov-type, [6]) in one as well as in several space dimensions [3, 5].

References

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