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An entropic kinetic scheme with compactly supported velocities

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We introduce a new kinetic model where the equilibrium distribution has a range of velocities centred about two discrete velocities ($\pm\lambda$) in 1D, as follows:

(1)
$$\mathbf{f}^{eq} = \begin{cases} f_1^{eq}, & \lambda - \Delta\lambda \leq v \leq \lambda + \Delta\lambda, \\ f_2^{eq}, & -\lambda - \Delta\lambda \leq v \leq -\lambda + \Delta\lambda, \\ 0, & \text{otherwise} \end{cases}$$

In 2D, the equilibrium distribution comprises of a range of velocities about four discrete velocities, one in each quadrant. Back in 1D, the moment relations for $\langle \mathbf{f}^{eq} \rangle$ and $\langle v\mathbf{f}^{eq} \rangle$, and the computed moment $\langle v^2\mathbf{f}^{eq} \rangle$ are used to formulate an equivalent discrete velocity Boltzmann equation (DVBE) like in [1].

(2)
$$\frac{\partial \mathbf{f}}{\partial t} + \frac{\partial (\widetilde{\Lambda}\mathbf{f})}{\partial x} = -\frac{1}{\epsilon}\left(\mathbf{f} - \widetilde{\mathbf{f}}^{eq}\right)$$

Here,$\widetilde{\Lambda}$ is a suitable diagonal matrix of ranges of velocities. The moments of the Boltzmann equation with BGK model, based on operator splitting of convection and collision steps and instantaneous relaxation to equilibrium ($\mathbf{f} = \widetilde{\mathbf{f}}^{eq}$), is used to obtain a kinetic scheme for the hyperbolic Euler system given by

(3)
$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{G}}{\partial x} = 0,\ \mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho E \end{bmatrix} \text{ and } \mathbf{G} = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ pu + \rho u E \end{bmatrix}$$

The essential difference is the range of velocities ($\lambda \pm \Delta\lambda$), which offer a significant advantage to obtain both an exact shock capturing as well as an entropic scheme. The range of velocities $\Delta\lambda$ serves the purpose of augmenting the stability of the model. $\Delta\lambda$ is used to provide additional diffusion in expansion regions. The average velocity λ is fixed to exactly capture grid-aligned steady discontinuities, by enforcing Rankine-Hugoniot jump conditions in the discretization process. Further, a novel kinetic relative entropy, appropriate to our discrete velocity model, is introduced which, along with an additional criterion, is used to identify expansions. Some standard 1D and 2D benchmark test cases are solved using this scheme.

Results: We solve some standard test cases for 1D and 2D Euler equations. In 1D, results are shown for steady contact-discontinuity, steady shock and unsteady Sod's shock tube problem test cases. For all these test cases, the domain is x $\in [0, 1]$, no of grid points, Nx $= 100$, and CFL=0.2. In 2D, first test case comprises of an oblique shock formed by a Mach 2.9 flow hitting the wall at 29 degrees. The next test case comprises of Mach 2 flow in a wind tunnel over a 15 degree ramp. These test cases involve the generation and further interaction of the nonlinear waves - shocks and expansions. The shock waves are captured crisply and the expansion waves are captured smoothly, without any unphysical features expected of low diffusion schemes.

References

[1] D. Aregba-Driollet and R. Natalini, Discrete kinetic schemes for multidimensional systems of conservation laws, *SIAM J. Numer. Anal.*, **37(6)** (2000), pp. 1973-2004

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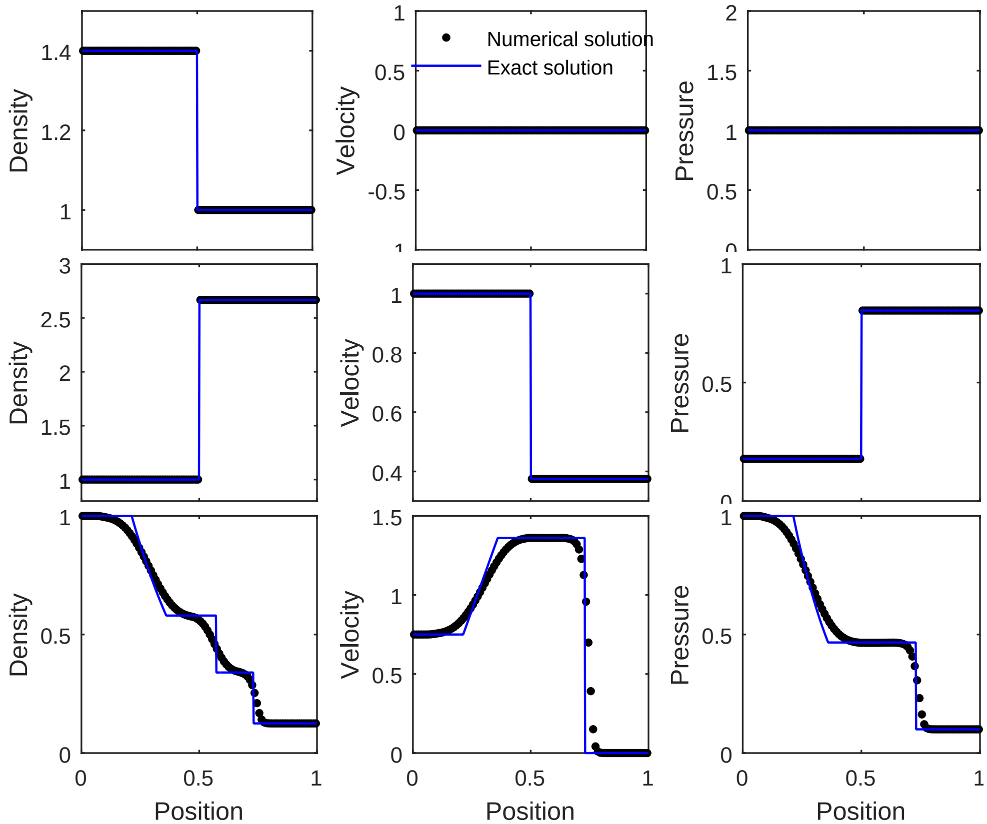

Figure 1: 1D Euler test cases: Top row) Steady contact- discontinuity; Middle row) Steady shock; Bottom row) Sod's shock tube problem, t=0.2

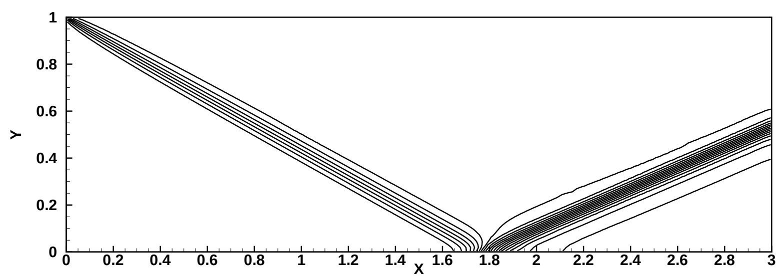

Figure 2: First order results for oblique shock reflection problem -Pressure contours(0.71:0.1:2.91) on a 240x 80 grid

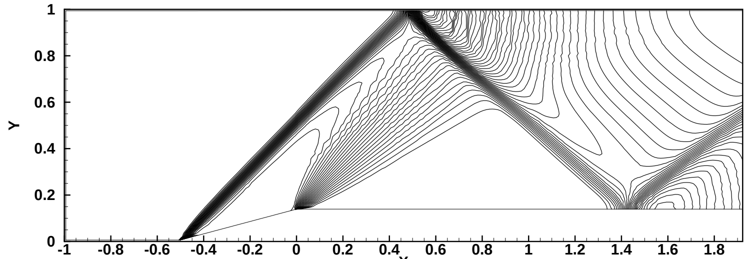

Figure 3: First order results for Mach 2 flow in a wind tunnel over a 15^o ramp -Pressure contours(1.1:0.05:3.8) on 240x 80 grid