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Central-Upwind Schemes and Contact Discontinuities

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The talk will be focused on central-upwind schemes, which are simple, efficient, highly accurate and robust Godunov-type finite-volume methods for hyperbolic systems of conservation laws. I will first briefly go over the derivation of the central-upwind schemes. The main steps are the following ones. First, we assume that the computed solution, realized in terms of its cell averages, is available at a certain time level. We then use a piecewise polynomial reconstruction, needed to increase the order of approximation, followed by the time evolution according to the integral form of the studied hyperbolic system. The evolution is performed using a nonsymmetric set of control volumes, whose size is proportional to the local speeds of propagation: this allow one to avoid solving any (generalized) Riemann problems, which makes the scheme, in fact, central (its upwind nature is in the fact that the eigenvalues of the Jacobian are used to determine both the size and shape of the control volumes. After the solution is evolved, it must be projected back onto the original grid as otherwise the number of evolved cell averages would double every time step and the scheme would become impractical. The projection should be carried out in a very careful manner as the projection step may bring an excessive amount of numerical diffusion into the resulting scheme as was the case in original central-upwind schemes introduced in [1, 2]. In order to more accurately project the solution, we had tried in [3] to use a sharper piecewise linear reconstruction of the evolved (intermediate) solution. This helped to reduce the amount of excessive numerical diffusion, but improvement in the resolution, especially in the resolution of linearly degenerate contact waves was not significant.

I will introduce a new way of making the aforementioned projection. A major novelty of the new approach is that instead of using a sharper piecewise linear reconstruction, we now use a subcell resolution and reconstruct the solution at each cell interface using two linear pieces. This allows us to perform the projection in the way, which would be extremely accurate in the vicinities of contact waves. As a result, the new second-order semi-discrete central-upwind scheme clearly outperforms its counterpart from [3] as confirmed by a number of numerical experiments conducted for both the 1-D and 2-D Euler equations of gas dynamics.

I will also show how the new central-upwind numerical flux can be utilized to construct a new finite-difference fifth-order finite-difference A-WENO scheme, which also outperfoms its A-WENO counterparts [4, 5], which are based on less accurate central-upwind numerical fluxes.

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References

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