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Well-Balanced High-Order Discontinuous Galerkin Methods for Systems of Balance Laws

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This work introduces a general technique to design high-order well-balanced Discontinuous Galerkin (DG) numerical schemes for systems of balance laws. In our approach a local projection step guarantees the exactly well-balanced character of the resulting numerical method for smooth stationary solutions. The strategy can be adapted to several time discretizations. Particularly, in this work, Runge–Kutta DG and ADER DG methods are studied. Additionally, a limiting procedure based on a modified WENO approach is described to deal with the spurious oscillations generated in the presence of non-smooth solutions, keeping the well-balanced properties of the scheme intact. The resulting numerical method is then exactly well-balanced and high-order in space and time for smooth solutions. Finally, some numerical results are depicted using different systems of balance laws to show the performance of the introduced numerical strategy.

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