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Well-balanced implicit-explicit Lagrange-projection-type schemes for the one-dimensional shallow water system

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In this work we present first and second order Lagrange-projection finite volume schemes for the shallow water equations. The Lagrange-projection scheme can be interpreted as a two-step algorithm consisting in first solving the shallow water system in Lagrangian coordinates, which is known as the Lagrangian step, and then projecting the results in Eulerian coordinates, which is known as the Projection step. This strategy allows us to decouple the acoustic and transport phenomena and to design implicit-explicit and large time step schemes in which the CFL restriction is based on the slower transport waves and not on the acoustic ones. In this work we follow the strategy described in [1,4] to define the LP scheme and [2] to ensure its well-balanced character.

For the Lagrangian step we propose two explicit versions, one first order and another one second order of accuracy. We also propose first and second order of accuracy implicit versions of the scheme. In the implicit version for the non-flat topography case, we give two different types of schemes: treating the source term explicitly and treating it implicitly which results in a nonlinear system that has to be solved. The projection on the Eulerian coordinates will always be done explicitly, preserving the total order of the scheme. Special care is taken for ensuring the well-balanced properties of the scheme.

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