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Stability analysis of microscopic models for traffic flow with lane changing

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In this talk we deal with microscopic modeling of traffic flow, focusing on lane changing dynamics. In particular we study a second order model that combines two different interaction terms.

We start from the description of the model in the two-lane case for a homogeneous population of $N \in \mathbb{N}$ vehicles. The dynamical equations of the system are obtained combining the term related to the Bando's model [1, 2] and the classical Follow-the-Leader interaction term [3, 4].

We propose lane changing conditions based according essentially on two criteria: incentive and security. A vehicle may change lane if it would travel at a faster speed in the new lane, which means that is would have a higher acceleration; and also the changing action must be safe in order to avoid collisions with the vehicles in the adjacent lane.

We denote by $x_n = x_n(t)$ and $v_n = v_n(t)$ the position and the velocity of the n-th vehicle at time $t \in \mathbb{R}^+$ and by $I_j(t) = I_j$ the set of indices of vehicles ordered by their position in lane j at time t. Thus the model can be written for $j = 1, 2$ as

$$(1) \qquad \begin{cases} \dot{x}_n = v_n \\ \dot{v}_n = \alpha(V_j(\Delta x_n^j) - v_n) + \beta \dfrac{\Delta v_n^j}{(\Delta x_n^j)^2} \quad n \in I_j \\ + \text{ lane changing conditions} \end{cases}$$

where $V_j(\cdot)$ is the desired velocity function for lane $j = 1, 2$. Assuming that vehicle n is currently in lane j then $\Delta x_n^j = x_{s_n^j} - x_n$ and $\Delta v_n^j = v_{s_n^j} - v_n$ denote the difference of positions and the difference of the velocities between vehicle n and its successive in the same lane. This construction can be easily generalized with a generic number of lanes.

Studying the stability of the steady states of this model in the one-lane case in which all vehicles are equally spaced and travel with the same constant velocity, we have found that the contribution of the parameter β is such that the stability region of this model is larger than the analogous stability region of Bando's model where $\beta = 0$.

The global steady state of the multi-lane model is parametrized by the total number N of vehicles in the road. All lanes are coupled by the lane changing conditions, and the equilibrium is reached only when the crowding of each single lane is such that no lane changing is convenient anymore. At that point the system can reach the equilibrium lane by lane.

In this setting we have proved that it is possible to determine conditions on perturbations in which the equilibrium of the steady state is preserved and lane changing does not occur.

We plan to derive a macroscopic version of this model where each lane would be described by its own equation and the lane changes would appear as source terms for the macroscopic hyperbolic equations. This study can be useful in applications for instance in the design of velocity profiles to minimize lane changes in order to avoid jams and car accidents.

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