XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

Nonlocal dynamics on graphs

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The talk is based on a recent study on nonlocal partial differential equations on graphs, and their continuum limits. Part of the motivation comes from applications to machine learning. Our main interest is in equations posed on graphs whose vertices are random samples of some underlying distribution and whose edge weights are a function of distances between vertices. In a recent paper, [2], we focus on dynamics driven by interaction energies on graphs. More precisely, in [2] we introduce graph analogues of the continuum nonlocal-interaction equation and interpret them as gradient flows with respect to a graph Wasserstein distance, using the Benamou–Brenier formulation, [1]. The graph continuity equation uses an upwind interpolation to define the density along the edges; while this approach has both theoretical and computational advantages, the resulting distance is only a quasi-metric. We investigate this quasi-metric both on graphs and on more general structures where the set of "vertices" is an arbitrary positive measure. We call the resulting gradient flow of the nonlocal-interaction energy the nonlocal nonlocal-interaction equation (NLNLIE), and develop existence theory as curve of maximal slope. Furthermore, we establish a discrete-to-continuum convergence result with respect to the number of vertices. In a forthcoming work, [3], we further investigate nonlocal dynamics on graphs considering a class of possible interpolations of the densities along the edges, and velocity fields depending on the mass configuration. By means of a fixed-point argument we can show existence and uniqueness of strong solutions.

Acknowledgements

This research has been supported by: EU-funded Erasmus Mundus programme "MathMods - Mathematical models in engineering: theory, methods, and applications" at the University of L'Aquila; German Science Foundation (DFG) through CRC TR 154 "Mathematical Modelling, Simulation and Optimization Using the Example of Gas Networks"; DFG under Germany's Excellence Strategy EXC 2044 – 390685587, *Mathematics Münster: Dynamics–Geometry–Structure,* and EXC 2047 – 390685813, *Hausdorff Center for Mathematics*; Collaborative Research Center 1060 – 211504053, *The Mathematics of Emergent Effects* at the Universität Bonn; NSF via grants DMS 1516677 and DMS 1814991 and via Ki-Net (SF Research Network Grant RNMS 1107444).

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