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Numerical solution of scalar conservation laws using semi-implicit WENO scheme

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In this talk, we deal with the numerical solution of scalar conservation laws using a novel semi-implicit time discretization combined with WENO (Weighted Essentially Non-Oscillatory) approximation in space [1]. The method can be viewed as a high-order extension (or alternative) to the 2nd order accurate Crank-Nicolson type of time discretization. The method can be explained easily by using a partial Cauchy-Kowalevski (or Lax-Wendroff) procedure where the time derivatives in Taylor series are replaced by the mixed derivatives of the solution exploiting the partial differential equations [2, 3]. Moreover, we use the inverse Lax-Wendroff procedure to approximate the boundary conditions [4].

We present the details of the third-order accurate scheme in the case of smooth solutions that can be viewed as an extension of the method in [5]. To avoid unphysical oscillations in the case of nonsmooth solutions, the third-order accurate scheme is expressed as a linear combination of two second-order accurate schemes, where the parameter of the combination is chosen depending on the smoothness of the numerical solution. The scheme is applied to some representative one-dimensional nonlinear conservation laws.

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