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Upwind methods for advection dominated level set equation with small curvature term

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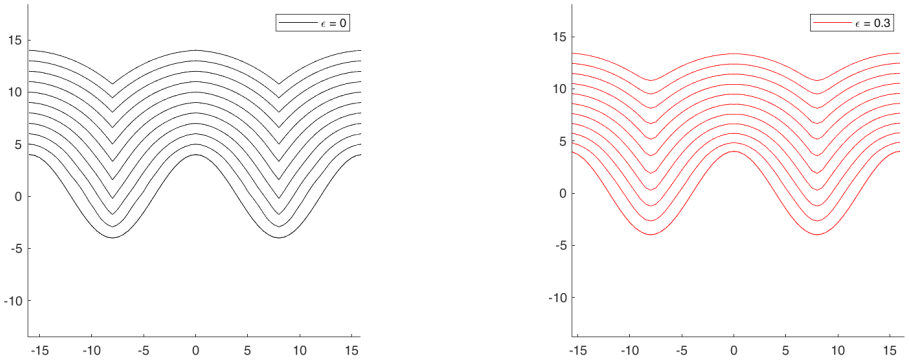
In this talk, we present new upwind based numerical schemes for advection dominated level set equation in two dimensions such as the eikonal equation supplemented with a small curvature term (1). The upwind stencils are proposed not only for the advection, but also for the approximation of second-order spatial derivatives in the curvature term. This approximation proofs beneficial when combined with fast sweeping [2] or fast marching [3] methods. To solve the discrete equations, one requires only a finite number of iterations that does not increase with the mesh refinement.

$$v\left(1-\epsilon\kappa\right)\left|\nabla\Phi\right|=1 \qquad \kappa=\nabla\cdot\left(\frac{\nabla\Phi}{\left|\nabla\Phi\right|}\right) \tag{1}$$

The proposed scheme might be used to solve (1), where the curvature κ is multiplied by a small regularization coefficient ϵ, compare the results in Figure 1. Such a model is used, e.g., to describe the movement of a forest fire front where the regularization term provides more appropriate physical accuracy, see Figure 2.



(a) Solution for $\epsilon = 0$ (b) Solution for $\epsilon = 0.3$

Figure 1: A comparison of two solutions for the equation (1) with $v = 1$ and space discretization step $h = 0.5$ where the bottom curve is fixed in both cases.

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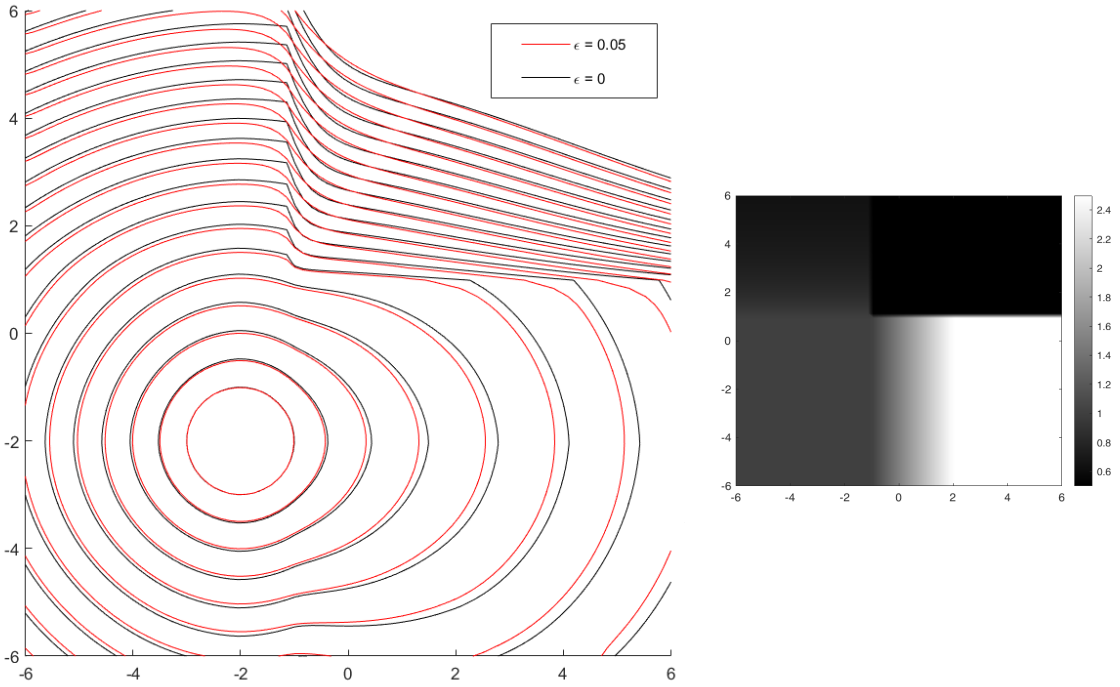

Figure 2: Two numerical solutions of forest fire front propagation modeled by the equation (1) where $v = v(x, y)$ (right picture) represents a variable combustibility of the vegetation. Solutions for $\epsilon = 0$ and $\epsilon = 0.05$ are presented for a comparison.

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