**XVIII International Conference on Hyperbolic Problems**
**Theory, Numerics, Applications**
**Málaga, June 20-24, 2022**

# Physics-informed neural networks and uncertainty quantification for hyperbolic transport models: Application to epidemic dynamics

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When studying real physical, biological, social dynamics by means of numerical techniques, the model parameters, necessary to simulate the forecasting scenarios, require a delicate calibration phase, often made even more challenging by the scarcity of available observed data. In addition, input data suffer inevitably from a large uncertainty, due to the stochasticity inherent in any measurement. Thus, we are commonly forced to draw conclusions and make decisions having only partial and random information at our disposal.

This is even more true when considering epidemic transport dynamics, as demonstrated by the recent COVID-19 pandemic especially in its early phase [1]. Indeed, observed data, required for a correct model calibration and to obtain reliable epidemic forecasts, are both limited and affected by a large uncertainty. Consider, for example, the real number of infected during the first wave of COVID-19 and the amount reported, instead, by official sources.

In this context, on the one hand, the use of Machine Learning techniques, in particular physics-informed neural networks (PINNs) [9, 10], to solve the corresponding inverse and forward problem of learning and data-driven solution of PDEs, respectively, turns out to be of paramount importance. On the other hand, the need for efficient methods capable of quantifying uncertainty in the mathematical models considered is essential in order to reproduce realistic scenarios.

In this presentation, a class of asymptotic-preserving (AP) PINNs for hyperbolic transport models with diffusive scaling are presented [4, 8]. A particular focus concerns their application to study the spatial propagation of infectious diseases, which is characterized by movements at different scales through the adoption of kinetic equations: a transport dynamics in the extra-urban scale, and a diffusive behavior in urban areas [2, 6, 7]. An example of results obtained with the AP PINNs compared with experimental data of the temporal evolution of infected individuals $I(t)$ and epidemic reproduction number $R_t$ is shown in Figure 1.
Furthermore, two different uncertainty quantification methods able to quantify the randomness in the investigated outputs are discussed: the first one based on the application of a stochastic Collocation Method [5] and the second one following a Bi-fidelity approach [3].

A series of numerical tests confirm the validity of the proposed approaches, including a realistic study of the outbreak of COVID-19 in Italy and its spread in the Lombardy Region.

## Acknowledgements

The support by GNCS–INdAM is acknowledged. This work is the result of different joint researches with L. Liu (The Chinese University of Hong Kong), C. Lu (Iowa University), L. Pareschi (University of Ferrara), and X. Zhu (Iowa University).

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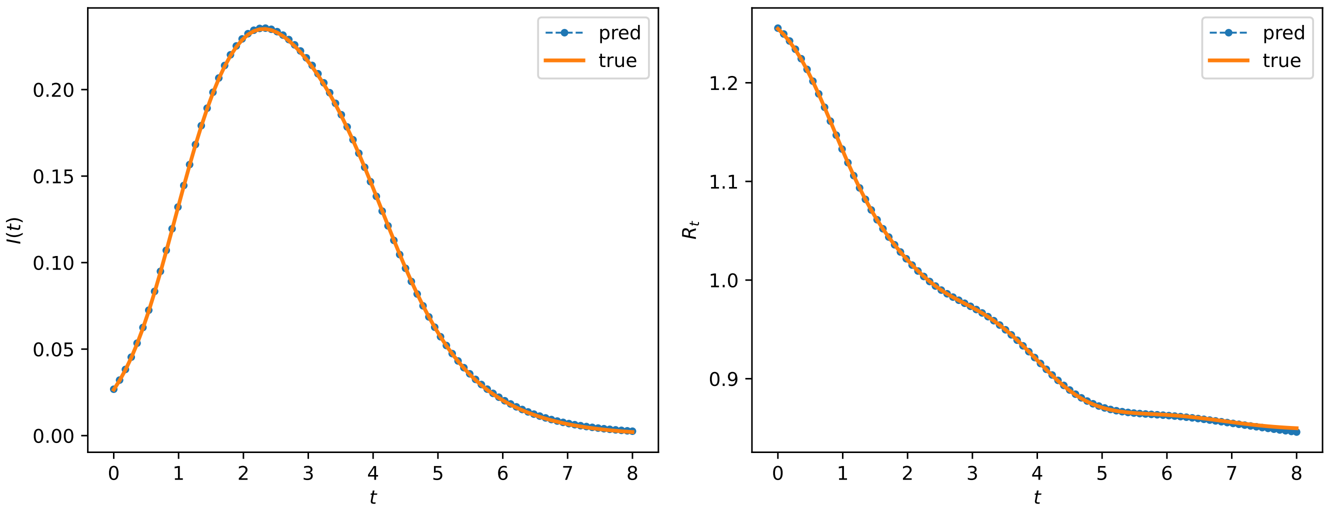


Figure 1: Results obtained with AP PINNs compared with experimental data of the temporal evolution of infected individuals $I(t)$ (left) and epidemic reproduction number $R_t$ (right).

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