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A posteriori Error Estimates for Numerical Solutions to Hyperbolic Conservation Laws

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Consider the Cauchy problem for a strictly hyperbolic system of conservation laws in one space dimension

$$u_t + f(u)_x \;=\; 0, \qquad u(0,x) \;=\; \bar{u}(x).$$

Assuming small total variation, it is well known that there exists a unique entropy-weak solution, depending continuously on the initial data. A priori estimates on the ^{1} distance between an approximate solution and the exact solution have been obtained in connection with (i) front tracking approximations, (ii) the Glimm scheme, and (iii) vanishing viscosity approximations. However, no a priori estimate is yet known for approximate solutions obtained by fully discrete schemes, such as the Lax-Friedrichs or the Godunov scheme. Therefore we focus on *a posteriori* error estimates. The result we show is the following.
Let u^{approx} be an approximate solution produced by a conservative scheme which dissipates entropy, and assume that

(i) the total variation of $u^{approx}(t,\cdot)$ is uniformly bounded,

(ii) outside a finite number of narrow strips in the domain $[0,T]\times\mathbb{R}$, the local oscillation of u^{approx} remains small.

Then the $\mathbf{L}^1$ distance

$$\|u^{approx}(T,\cdot) - u^{exact}(T,\cdot)\|_{\mathbf{L}^1(\mathbb{R})}\,.$$

is small. Our estimates do not require any regularity of the exact solution. We provide an error bound which can be applied to a wide class of approximation schemes [1].

References

[1] A. Bressan, M.T. Chiri and W. Shen, A posteriori Error estimates for numerical solutions to hyperbolic conservation laws. *Arch. Rat. Mech. Anal.* , **241** (2021), 357–402

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