XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

# On the hyperbolicity region of Rational Extended Thermodynamics models for rarefied gases

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Rational Extended Thermodynamics (RET) is a well-known phenomenological field theory capable of describing non-equilibrium phenomena and rapid changes in space-time out of local equilibrium [1, 2]. The theory is constructed starting from the validity requirement of universal principles, such as the objectivity principle and the entropy principle. This gives the theory a particularly elegant and robust structure both from the mathematical and the physical standpoint. In fact, the RET models are expected to be symmetric hyperbolic PDE systems with a convex entropy, so that the well-posedness of the Cauchy problem is guaranteed. The hyperbolicity property is also very important for a realistic physical description, since it is associated with finite speeds of disturbances, in contrast with the infinite speed predicted by the parabolic models of Classical Thermodynamics of Irreversable Processes. Usually, RET is considered in a neighbourhood of an equilibrium state and the corresponding partial differential equations are linear with respect to the non-equilibrium variables, providing systems of Grad's type. Consequently, also the hyperbolicity condition remains valid only in a certain domain of the state variables (called *hyperbolicity region*). The analysis about the determination of such a region was started by Müller and Ruggeri more than 25 years ago.

The aim of this talk is to present some very promising results in the case of rarefied monatomic and polyatomic gases (polytropic and non-polytropic) [3, 4]. We construct second-order RET theories, following the ideas introduced in [5], and we show that such models present a larger hyperbolicity region compared to that of the first-order approximation.

## Acknowledgements

This research was supported by Gruppo Nazionale di Fisica Matematica (GNFM) dell'INdAM and in part (F.B.) by the Italian research Project PRIN 2017 No. 2017YBKNCE "Multiscale phenomena in Continuum Mechanics: singular limits, off-equilibrium and transitions."

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