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Optimal Control of Traffic in Road Networks

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In this talk we are interested in the control of a continuous road traffic model, applied to crisis management involving vehicles in an urban environment.

The problem is formalized using an oriented graph where the edges are the roads and the vertices are the junctions. If the fluid model describing the flow is very standard (Lighthill-Whitham-Richards, 1955), the flow distribution problem at junctions realizes a nonlinear coupling between the different edges, inspired by recent works [1],[3].

Thus, the distribution of vehicles at junctions is modeled by an optimal redistribution process depending on the maximum achievable flows at the junctions, via a linear programming problem aiming at maximizing the flows. In order to provide a means of influencing the traffic flow, we introduce control functions defined at each road entrance, acting as a barrier by weighting the capacity of a road leaving a junction to receive new vehicles. We denote by $\boldsymbol{u}$ the vector of control functions. See [2] for another approach of this problem.

The evolution on roads $[a_i, b_i]$ of the densities ρ_i subjected to the control $\boldsymbol{u}$ is given by the following system:

$$(1) \qquad \begin{cases} \partial_t \rho_i(t,x) + \partial_x f_i(\rho_i(t,x)) = 0, & (t,x) \in (0,T) \times [a_i, b_i], \\ \rho_i(0,x) = \rho_i^0(x), & x \in [a_i, b_i], \\ f_i(\rho_i(t,a_i)) = \gamma_i^L(t, \rho(t), \boldsymbol{u}(t)), & t \in (0,T), \\ f_i(\rho_i(t,b_i)) = \gamma_i^R(t, \rho(t), \boldsymbol{u}(t)), & t \in (0,T), \end{cases}$$

where the functions γ^L and γ^R are the solutions of the linear programming problem at the edges of each route, and ρ^0 denotes the initial condition.

In this talk, we will focus on the following problem: given a graph, a main road and knowing the vehicle flows at any point, how can we use the control to empty the road in a fixed time? We model this question using an optimal control problem consisting in minimizing the density in final time on a predefined graph path, i.e. minimizing the functional :

$$(2) \qquad J(\boldsymbol{u}) = \sum_{i \in \text{path}} \int_{a_i}^{b_i} \rho_i(T, x; \boldsymbol{u})\, dx,$$

under the following constraints:

- $0 \leq u_i(t) \leq 1$ which reflects the fact that the control is a vehicle acceptance rate on a road,
- $\sum_{i \in \text{ routes}} u_i(t) \leq N_{\text{max}}$ which translates that we imposes a maximum number of active controls at a given time (to take into account the staff used to make the roadblocks for example).

The resulting problem being strongly non-linear and of large size, obtaining an efficient numerical method faces various difficulties, relating to the computation time, the presence of many local minima and the difficulty to make standard algorithms converge.

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We propose and test an *"optimize-then-discretize"* algorithm coupling a classical descent method involving adjoint state with a fixed-point method based on optimality conditions. We apply this method to complex graphs where evacuation scenarios can be studied.

References

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