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Semi-implicit numerical methods for conservation laws and level set problems

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Implicit type of time discretization methods for conservation laws has been continuously considered in numerical analysis, see [1, 2, 3, 4, 5] for some recent developments. In this talk, we present a class of semi-implicit methods for some representative partial differential equations used in conservation laws or in level set methods. Previously, one-parameter class of numerical schemes was derived for the linear advection equation with variable velocity in nonconservative [6, 7] and conservative form [7]. There, several advantages of the scheme could be clearly shown and illustrated with respect to some standard implicit schemes. Furthermore, some variants of this method were successfully applied to the nonlinear advective level set equation [8] including polyhedral meshes [9]. In this talk, we consider their extension for nonlinear conservation laws.

The considered parametric class of semi-implicit methods with unconditional stability is derived as a high order accurate extension of the first order implicit time discretization method presented in [3] for conservation laws. The method is formally based on the partial Cauchy-Kowalevski (or Lax-Wendroff) procedure where the time derivatives in Taylor series are replaced by mixed derivatives exploiting the partial differential equations. This is a main modification of the standard procedure in which the time derivatives are replaced by strictly spatial derivatives. Using some appropriate approximations of the mixed derivatives results in a convenient form of the Jacobian for nonlinear algebraic equations (or the matrix in the case of linear equations) when the system of equations can be solved efficiently using e.g. a fast sweeping strategy [3].

In this talk, we present the high order extension of the method in [6, 10] for linear and nonlinear scalar conservation laws where the free parameter of the method is used to suppress unphysical oscillations for nonsmooth solutions using some standard and newly developed TVD limiters and ENO/WENO strategies.

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