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Homogenization of Stochastic Conservation Laws with Multiplicative Noise

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We consider the generalized almost periodic homogenization problem for two different types of stochastic conservation laws with oscillatory coefficients and multiplicative noise. Namely,

$$du^\varepsilon + a\left(\frac{x}{\varepsilon}\right) \cdot \nabla_x f(u^\varepsilon)\, dt = \kappa_0\, \sigma(u^\varepsilon) \circ dW, \tag{1}$$

and

$$du^\varepsilon + \nabla_x \cdot f(u^\varepsilon)\, dt = \frac{1}{\varepsilon} V'\left(\frac{x_1}{\varepsilon}\right) dt + \kappa_0\, \sigma_{f_1}(u^\varepsilon) \circ dW,$$

where dW is a Brownian motion, $\circ dW(t)$ denotes integration in the sense of Stratonovich for the stochastic integral and a and V belong to some ergodic algebra $\mathcal{A}(\mathbb{R}^d)$ (e.g. a and V being periodic or almost periodic functions).

In both cases the stochastic perturbations are such that the equation admits special stochastic solutions which play the role of the steady-state solutions in the deterministic case.

Specially in the second type, these stochastic solutions are crucial elements in the homogenization analysis.

Our homogenization method is based on the notion of stochastic two-scale Young measure, whose existence is established here.

As far as we are know, the existence of two-scale Young measures had not been proven so far in this stochastic setting. We show that, under certain integrability conditions, a sequence of appropriately bounded predictable functions generates a stochastic two-scale Young measure that characterizes the two-scale convergence of a subsequence. Moreover, we show that the limit is predictable and that the stochastic two-scale Young measure commutes with the stochastic integral.

With this result at hand, in both stochastic homogenization problems that we consider, we use the special stochastic solutions in order to bound the sequence of solutions appropriately so that they generate a two-scale stochastic Young measure which we use to identify the homogenized equation that characterizes the weak limit of the sub-sequence. Then, we use a rigidity result, which we also prove, to show the consistency of this limit, thus generalizing to the stochastic setting the results available for the homogenization of these equations in the deterministic case (i.e. $\sigma = 0$ and $\sigma_{f_1} = 0$).

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