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A correction of the friction term in depth-averaged granular flow models related to the motion/stop criterion

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Because of their simplicity and low computational cost, depth-averaged shallow models are often used to describe gravitational granular flows at the laboratory and field scales, based on the pioneer model proposed by Savage & Hutter [1]. To impose this approximation, local coordinates are introduced, associated to a reference plane following the slope, and an averaging of the governing equations is performed in the direction perpendicular to this plane.

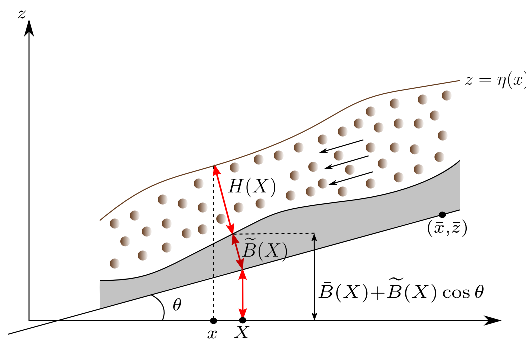

Figure 1: Topography and the granular layer at time t

In this talk, we focus on how shallow models tackle the threshold of motion in granular flows over slopping beds and we highlight key misleading behaviour of such models. To see this, let us consider the classical Savage-Hutter model (1D) written in local coordinates as:

$$
(1) \qquad \begin{cases} \partial_t H + \cos(\theta)\, \partial_X(HU) = 0, \\ \partial_t(HU) + \cos(\theta)\partial_X(HU^2) = -gH\cos(\theta)\Big(\partial_X \bar{B} + \partial_X(\widetilde{B} + H)\, \cos(\theta) + \mathrm{sgn}(U)\mu_b\Big), \end{cases}
$$

where $U = U(X,t)$ is the velocity component parallel to the reference plane and μ_b is the basal friction coefficient. In that model, the interaction between the granular material and the bottom surface is determined by the basal friction force. The corresponding stress appears in the momentum equation as a term depending on the basal friction coefficient μ_b multiplied by the normal stress $gH\cos(\theta)$. This term not only defines the friction effect during the flow but it is responsible of an essential task intrinsic to this

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problem which is to provide a threshold of motion for the Coulomb criterion. Thus, for the simplest case of a constant basal friction coefficient $\mu_b = \tan\delta_0$, the physical threshold of motion for the material initially at rest is given by the repose angle δ_0, measured experimentally as the limit slope of the free surface of a pile of the granular material at rest. As a result, the material stays at rest when:

$$|\partial_x \eta(x)| \leq \tan\delta_0, \tag{2}$$

where $\eta(x)$ denotes the free surface. On the other hand, a look at the momentum equation of the above system reveals that the solution of the model stays at rest when the driving forces do not exceed the friction term, that is,

$$|\underbrace{\partial_X\big(\bar{B} + (\widetilde{B} + H)\,\cos(\theta)\big)}_{\partial_X \eta(X)}| \leq \mu_b (= \tan(\delta_0)). \tag{3}$$

Since the threshold criteria (3) should correspond to the physical criteria (2), then we would have $|\partial_X \eta(X)| = |\partial_x \eta(x)|$. As we will show, this is not true for any definition of the local coordinates. In fact,

$$\partial_X \eta(X) = \mathcal{J}\ \partial_x \eta(x). \tag{4}$$

where $\mathcal{J} = 1 - \partial_X\big(\widetilde{B}(X) + H(X)\big)\sin(\theta)$. This is why we propose to set $\mathcal{J}\mu_b$ instead of μ_b as the friction coefficient in the momentum equation above.

We also extend the model and the modification of the motion criterion to granular flows on 2D topography and, finally, some numerical results highlighting the problem related to the threshold of motion and how the modified criterion affects the simulations are presented. The results and the limitation of the proposed modification are discussed based on comparison with laboratory experiments.

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