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Equivalent equation analysis of a kinetic relaxation model

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We consider the scalar conservation law in 2 dimensions

$$(\mathcal{E}) \qquad \partial_t w + \nabla \cdot \boldsymbol{q}(w) = 0.$$

To solve numerically this equation, we use a kinetic relaxation scheme $D2Qn_v$ which approximate it with n_v equations and n_v variables.

Kinetic models have the advantage to be efficient numerical scheme which use transport at constant velocities. However, it can be difficult to analyze them.

To study this method, we propose to approximate the solution given by the kinetic model by an equivalent equation with n_v variables: the unknown w of our initial equation and $n_v - 1$ additional variables.

In one dimension, the equivalent equation of the $D1Q2$ kinetic model has been studied in [1] and gives second-order boundary conditions. The analysis of the equivalent equation also gives us information on the stability: it allows us to define a condition to achieve the hyperbolicity of the kinetic model, that we can compare to the subcharacteristic stability condition.

We apply this kinetic relaxation method to a transport equation from plasma physic, the guiding-center model

$$\begin{cases} \partial_t \rho + \boldsymbol{v} \cdot \nabla \rho = 0, \\ -\Delta \phi = \rho, \end{cases}$$

where ρ is the ion density, ϕ is the potential and E is the electric field such as $\boldsymbol{v}(\boldsymbol{x}, t) = (-\nabla \phi(\boldsymbol{x}, t))^{\perp} = E(\boldsymbol{x}, t)^{\perp}$.

Usually, this model is solved with a semi-Lagrangian scheme. But, we choose to use the structure of the kinetic equations to build a CFL-less Discontinuous Galerkin scheme, which does not require inversion of matrices.

For example, the Diocotron testcase gives us the densities of Figure 1 at time $t = 80$, $t = 90$ and $t = 100$, with a $D2Q4$ kinetic scheme.

References

[1] F. Drui, E. Franck, P. Helluy, and L. Navoret. *An analysis of over-relaxation in a kinetic approximation of systems of conservation laws.* Comptes Rendus Mécanique , 347(3):259-269, 2019.

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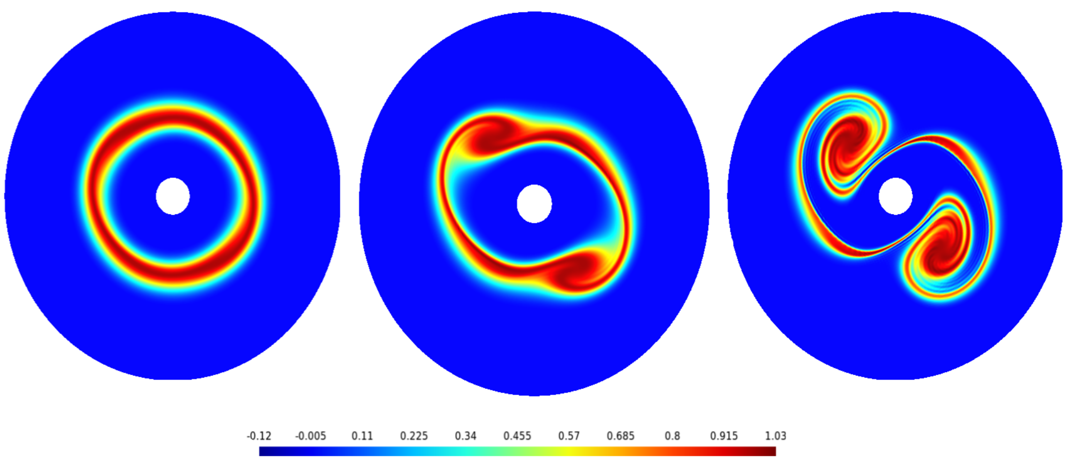

Figure 1: Densities obtained for the Diocotron testcase at time $t = 80$, $t = 90$ and $t = 100$.