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The implicit Lagrangian Riemann problem

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The implicit discrete formulation of non linear equations that arise in gas dynamics problems appeared in the seminal works of Godunov [2]. However as always with non-linear continuous or discrete [1] equations, existence and uniqueness of the solutions are not guaranteed.

In our work with A. Plessier and S. Del Pino, we address the mathematical and numerical structure of implicit Riemann solvers for Lagrangian equations. We discovered that one particular version based on a splitting between an isentropic prediction followed by an energy-conservation correct is endowed with very strong convex structure. On this basis we proved existence and uniqueness of the implicit solution, which seems to be an original result [3]. Moreover we discovered that the scheme is extremely efficient for tracking the position of the contact discontinuity, quasi-uniformly with respect to the time step $\Delta t > 0$ which can be taken very large. An explanation of this efficiency is proposed by exact integration of the implicit isentropic Lagrangian problem (discrete in time, continuous in space) with Riemann initial data

$$\left\{ \begin{array}{l} \frac{\tau(x)-\tau_\pm}{\Delta t} - \partial_m u = 0, \\ \frac{u(x)-u_\pm}{\Delta t} + \partial_m p = 0, \\ \frac{S(x)-S_\pm}{\Delta t} + \partial_m (pu) = 0, \end{array} \right.$$

I will also report on our current investigations on the 2D formulation.

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References

[1] Toro-Montecinos. Implicit, semi-analytical solution of the generalized Riemann problem for stiff hyperbolic balance laws, JCP 2015.

[2] Godunov et al, Numerical Solving Many-Dimensional Problems of Gas Dynamics, Nauka, 1976

[3] Del Pino-Després-Plessier, Implicit discretization of Lagrangian gas dynamics, `https://hal.archives-ouvertes.fr/CEA-UPSAY/cea-03284467v1`, 2021.

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