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Two-dimensional incomplete Riemann solvers for shallow water systems with topography and dry areas

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A second-order genuinely two-dimensional class of finite volume methods is proposed for hyperbolic nonconservative systems, with special attention given to the solution of shallow water systems with topography and dry areas. Two-dimensional features are included in the solver through the solution of 2D Riemann problems at the vertices of the computational cells, following the ideas in [1, 4, 5].

The methods are based on incomplete two-dimensional Riemann solvers, for which only a bound on the maximal speeds of propagation in the coordinate directions is needed. Moreover, the amount of numerical diffusion can be easily controlled by appropriate choices of the numerical viscosity matrices [2, 3, 4].

Departing from the schemes previously introduced by the authors in [5], which were unable to handle the appearance of dry areas in the computational domain, we have adapted them to correctly handle the existence of wet-dry transitions, which may be present in the initial condition or may appear as the fluid evolves. This is not a trivial task, as many difficulties appear when coupling the one- and two-dimensional fluxes arising at edges and vertices, respectively. On the other hand, the proposed schemes maintain their well-balancing properties also in the case of stationary solutions containing dry areas [6].

Several numerical tests including genuinely two-dimensional effects and the appearance of dry areas are presented to test the performances of the proposed schemes. An efficient GPU implementation allow us show that, in general, 2D Riemann solvers are more efficient than their 1D$\times$1D versions.

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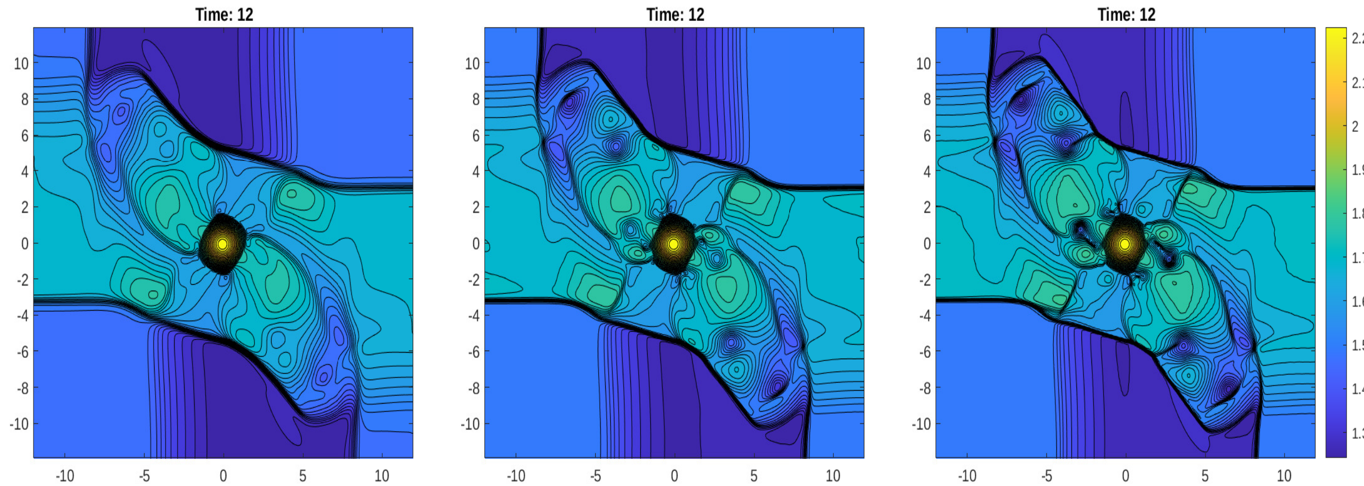

Figure 1: Test: 2D Riemann problem with a emerged bump. Countour plots of the interface on a 400×400 meshgrid. Left: 2D HLL scheme; center: 2D PVM scheme based on a polynomial of eight degree; right: 2D RVM scheme based on a rational function of of degree 8/8.

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