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Aggregation with intrinsic interactions on Riemannian manifolds

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We consider an aggregation model that consists in an integro-differential equation for the evolution of a population density $\rho(x,t)$ on a Riemannian manifold M:

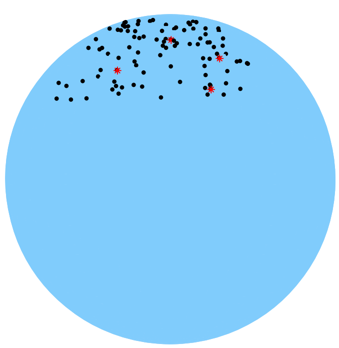
$$\rho_t + \nabla_M \cdot (\rho \mathbf{v}) = 0, \tag{1a}$$

$$\mathbf{v} = -\nabla_M K * \rho. \tag{1b}$$

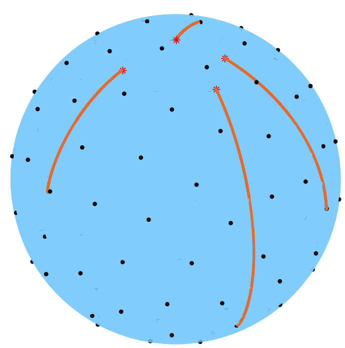
Here, $K : M \times M \to \mathbb{R}$ is an interaction (also known as aggregation) potential which models social interactions such as attraction and repulsion, and $\nabla_M \cdot$ and ∇_M represent the manifold divergence and gradient, respectively. The symbol $*$ denotes a measure convolution: for a time-dependent measure ρ_t on M and $x \in M$, we set

$$K * \rho_t(x) = \int_M K(x,y)\,\mathrm{d}\rho_t(y). \tag{2}$$

We study model (1) with *intrinsic* interactions, where the interaction potential depends on the geodesic distance on M between points. We establish the well-posedness of measure solutions, defined via optimal mass transport, and investigate the long-time behaviour of solutions on certain specific manifolds. The analytical results are illustrated with numerical experiments that exhibit various asymptotic patterns. This is joint work with H. Park, F. Patacchini, and B. Zhang.



(a)



(b)

Figure 1: A random initial configuration of particles (a) evolves into a uniform particle distribution over the sphere (b).

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