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3D shape reconstruction via Hamilton-Jacobi equations

S. TOZZA *

The three-dimensional reconstruction of an object is an interesting topic with many applications in different fields, and has attracted several researchers. The applications range goes from the biomedical 3D reconstruction of human tissues to the approximation of the surface of astronomical objects, from archeology for the digitization of artistic works to the recent development of 3D printing. The first being interested in this problem were some opticians in the Fifties-Sixties [1, 2]. Afterwards, B.K.P. Horn and his collaborators of MIT first formulated the Shape-from-Shading (SfS) problem for a single gray-level image of the object [3, 4, 5] . The goal was to get the 3D surface represented in the input image solving a partial differential equation or a variational problem. This problem gave rise to an expansion in the field of mathematics and some researchers tried to prove the well-posedness in the framework of weak solutions. The first works of Lions, Rouy and Tourin in the early 90s [6, 7] inserted the SfS problem in the context of the viscosity solutions frameworks, hence in a much more theoretical area. In this talk I will start dealing with the orthographic SfS problem with Lambertian reflectance model, the classical and simplest setup for this ill-posed problem, that can be modeled by first order Hamilton-Jacobi equations. I will continue exploring some non-Lambertian reflectance models [8] and we will see how it is possible to derive a well-posed problem adding information in a natural way, i.e. considering more than one input image [9, 10, 11]. Staying within the scope of a single input image, moving to a perspective projection can be useful, arriving to the well-posedness of the problem in the context of viscosity solutions [12, 13].

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*Dept. of Mathematics, ALMA MATER STUDIORUM - Università di Bologna, Italy. Email: silvia.tozza@unibo.it