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The Riemann solution for the generic three-phase flow of the Glimm-Isaacson model in porous media

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At the beginning of the eighties James Glimm and Eli Isaacson [4] considered the Riemann problem for a system of two conservation laws modeling a two-phase (water-oil) flow in a porous medium with a small amount of polymer dissolved in the water phase given by

$$\begin{cases} u_t + f(u,c)_x = 0 \\ (cu)_t + (cf(u,c))_x = 0, \end{cases} \tag{1}$$

where $x \in \mathbb{R}$, $t > 0$, $u(x,t)$ is the water saturation, $(1 - u(x,t))$ is the oil saturation, $c(x,t)$ is the concentration of the polymer in the water phase. The S-shaped flux function f in (1) is based on Corey's model for the permeabilities. It is given by $f(u,c) = \frac{u^2}{\mu_w(c)}/D(u,c)$, with $D(u,c) = \frac{u^2}{\mu_w(c)} + \frac{(1-u)^2}{\mu_o}$. Here $\mu_w(c)$ is the viscosity of the water phase, taken as a smooth increasing function of c, and μ_o is the constant viscosity of the oil phase.

The Riemann solutions of (1) were fully obtained and consist of concatenations of $c-$waves (contact waves), connecting two distinct polymer concentration levels and $u-$waves (saturation or Buckley-Leverett waves) at such constant $c-$levels.

Based on the Isaacson solution, in [11] we increased the complexity of the system (1) by adding a gas phase in the model obtaining the 3×3 conservation laws system

$$\begin{cases} \begin{cases} u_t + f(u,v,c)_x = 0 \\ v_t + g(u,v,c)_x = 0 \end{cases} & \text{(saturation system)}, \\ (cu)_t + (cf(u,v,c)_x = 0 & \text{(concentration equation)} \end{cases} \tag{2}$$

In system (2) we have the water phase saturation u, the oil phase saturation v and the gas saturation $(1 - u - v)$. The flux functions are given by $f(u,v,c) = \frac{u^2}{\mu_w(c)}/D(u,v,c)$ and $g(u,v,c) = \frac{v^2}{\mu_o}/D(u,v,c)$, with $D(u,v,c) = \frac{u^2}{\mu_w(c)} + \frac{v^2}{\mu_o} + \frac{(1-u-v)^2}{\mu_g}$.

The Riemann solutions obtained in [11] for system (2) had the constraint that the gas and oil phase viscosities were taken constant and equal, due to the fact that the Riemann solution for the 2×2 saturation system was known only in this situation [5, 10]. This restriction was overcome in the last decades and currently we are able to describe the solutions of the 2×2 saturation system, for more general cases in which the phase viscosities are considered to be highly different from each other. See [1, 2, 3] for instance.

In this work we return to the 3×3 system (2) taking into account these recent developments. Based on the wave curve method [1, 6, 7, 8] and aided by the computer code ELI [9] we can now construct Riemann solutions for unequal phase viscosities. We present Riemann solutions with right states representing mostly oil compositions and left states representing generic injection data with a water-gas mixture in a porous medium.

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