XVIII INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS
THEORY, NUMERICS, APPLICATIONS
Málaga, June 20-24, 2022

Existence of Global in Time Weak Solutions to Singular 3D Quasi-Geostrophic Systems

Yiran Hu [*], Alexis Vasseur [†]

In this talk, we show the existence of global in time weak solutions to a family of singular 3D quasi-geostrophic systems with Ekman pumping, where the background density profile is degenerate at the boundary. The 3D Quasi-Geostrophic system was studied in the book Quasi-Geostrophic Dynamics by Joesph Pedlosky[5]. They describe stratified flows in the atmosphere on large time scale and are widely used for forecasting atmospheric circulation. These systems couple an inviscid transport equation in 3D with an equation on the boundary satisfied by the trace. When there is no degeneracy, Puel and Vasseur [6] have proved the existence in L^2 when there is no viscosity and later Matthew Novack generated it to any L^p[3]. Moreover global well-posedness is studied for 3D system [4] as well as considering the boundary only [7] [1] [2] . The degeneracy near boundary brings difficulty in lost of ellipticity and trace theorem. But thanks to the Muckenhoupt class A_2 weighted space, we generalized some properties. The proof is based on the construction of approximated models which combine the Galerkin method at the boundary and regularization processes in the bulk of the domain. The solution to the QG system is obtained at the limit thanks to Aubin-Lions lemmas.

[*]University at Texas at Austin. Email: yrhu@math.utexas.edu
[†]University at Texas at Austin. Email: vasseur@math.utexas.edu

References

[1] Alexander Kiselev, Fedor Nazarov, and Alexander Volberg. Global well-posedness for the critical 2d dissipative quasi-geostrophic equation. *Inventiones mathematicae*, 167(3):445–453, 2007.

[2] Changxing Miao and Liutang Xue. Global well-posedness for a modified critical dissipative quasi-geostrophic equation. *Journal of Differential Equations*, 252(1):792–818, 2012.

[3] Matthew D Novack. On the weak solutions to the 3d inviscid quasi-geostrophic system. *arXiv preprint arXiv:1709.10495*, 2017.

[4] Matthew D Novack and Alexis F Vasseur. Global in time classical solutions to the 3d quasi-geostrophic system for large initial data. *Communications in Mathematical Physics*, 358(1):237–267, 2018.

[5] Joseph Pedlosky. *Geophysical fluid dynamics*. Springer Science & Business Media, 2013.

[6] Marjolaine Puel and Alexis F Vasseur. Global weak solutions to the inviscid 3d quasi-geostrophic equation. *Communications in Mathematical Physics*, 339(3):1063–1082, 2015.

[7] Serge G Resnick. Dynamical problems in non-linear advective partial differential equations. 1996.