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Provably Energy Stable Approximations of Nonlinear Hyperbolic Problems

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We present the general stability theory for hyperbolic IBVPs developed in [1]. It extends the use of the energy method to nonlinear problems, is easy to understand and leads to L_2 estimates. The only requirements for an energy bound is that a skew-symmetric form of the equations exist and that proper boundary conditions are available. A nonlinear and linear analysis may leads to different boundary conditions required for a bound [2]. The new formulation shed light on this confusing fact.

The new skew-symmetric formulation was derived for the shallow water equations [1] and the compressible Euler equations [3]. We will discuss how to determine nonlinear boundary conditions and relate that to a boundary condition analysis for linear problems. Finally, by discretising using summation-by-parts (SBP) operators [4] which mimic integration-by-parts, we show that nonlinear stability follows automatically.

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References

[1] J. Nordström. *Nonlinear and linearised primal and dual initial boundary value problems: When are they bounded? How are they connected?* Journal of Computational Physics, vol 455, No 111001, 2022.

[2] J. Nordström & Andrew R. Winters *Linear and nonlinear analysis of the shallow water equations* arXiv:1907.10713, 2021

[3] J. Nordström. *A new energy stable formulation of the compressible Euler equations.* arXiv:2201.05423v2, 2022.

[4] M. Svärd & J. Nordström *Review of Summation-By-Parts Schemes for Initial-Boundary-Value Problems.* Journal of Computational Physics, Volume 268, pp. 17-38, 2014.

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