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Numerical analysis for the shallow water model with two velocities

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The Shallow water equations (also called Saint-Venant's equations) are the usual model governing fluid flow in the rivers, channels or the oceans. They are used, for example, for the protection of the environment, the prediction of tides and storm urges, the transport of the sediment or the study of floods. Some references in the literature propose an improvement of the Shallow water equations to take into account the vertical profile of the horizontal velocity, see [2, 4].

The objective of this work is to develop a scheme of the model with two velocities in the vertical profil based on the analysis of the Riemann problem performed in [1]. We look for a scheme able to exactly recover any steady solution in 1D over arbitrary topography. To do so, first we analyse the moving steady solutions following the Bernouilli's principle for C^1 regular solutions and the Rankine-Hugoniot conditions as well as the dissipation of entropy at a point of discontinuity. We then propose several well-balanced and positivity preserving Riemann solvers following the strategy proposed in [3]. Finally, we validate our results with numerical simulations.

References

[1] AGUILLON, Nina, AUDUSSE, Emmanuel, GODLEWSKI, Edwige, et al. Analysis of the Riemann Problem for a shallow water model with two velocities SIAM Journal on Mathematical Analysis, 2018, vol. 50, no 5, p. 4861-4888.

[2] AUDUSSE, Emmanuel, BRISTEAU, Marie-Odile, PERTHAME, Benoît, et al. *A multilayer Saint-Venant system with mass exchanges for shallow water flows. Derivation and numerical validation.* ESAIM: Mathematical Modelling and Numerical Analysis, 2011, vol. 45, no 1, p. 169-200.

[3] MICHEL-DANSAC, Victor, BERTHON, Christophe, CLAIN, Stéphane, et al. *A well-balanced scheme for the shallow-water equations with topography.* Computers & Mathematics with Applications, 2016, vol. 72, no 3, p. 568-593.

[4] RICHARD, Gaël L. et GAVRILYUK, Sergey L. *A new model of roll waves: comparison with Brock?s experiments.* Journal of Fluid Mechanics, 2012, vol. 698, p. 374-405.

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