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High Friction limits for Euler-Korteweg and Navier-Stokes Korteweg models

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The aim of the paper [9] is to investigate the singular relaxation limits for the Euler–Korteweg and the Navier–Stokes–Korteweg system in the high friction regime. We shall prove that the viscosity term is present only in higher orders in the proposed scaling and therefore it does not affect the limiting dynamics, and the two models share the same equilibrium equation. The analysis of the limit is carried out using the relative entropy techniques in the framework of weak, finite energy solutions of the relaxation models converging toward smooth solutions of the equilibrium. The results proved here take advantage of the enlarged formulation of the models in terms of the *drift velocity* introduced in [7], generalizing in this way the ones proved in [17] for the Euler–Korteweg model. We studied also the case of multicomponent systems by adapting the results contained in [15] and [19] taking into account the two different time-scaling.

The relaxation-time limit from the Quantum Navier-Stokes-Poisson system to the quantum drift-diffusion equation is performed in [1] in the framework of finite energy weak solutions. No assumptions on the limiting solution are made. The proof exploits the suitably scaled a priori bounds inferred by the energy and BD entropy estimates. Moreover, it is shown how from those estimates the Fisher entropy and free energy estimates associated to the diffusive evolution are recovered in the limit. As a byproduct, our main result also provides an alternative proof for the existence of finite energy weak solutions to the quantum drift-diffusion equation.

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