XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

On a General Non-hydrostatic Formulation for Boussinesq Dispersive Shallow Water Systems and its Numerical Approximation

Cipriano Escalante Sánchez [*], Tomás Morales de Luna [†]

This work proposes a general framework that collects many of the most well-known Boussinesq-type systems used for coastal type applications. Within such formulation, dispersive shallow water systems are rewritten, avoiding using high-order derivatives that are not easy to treat numerically [3].

These PDE dispersive systems of hyperbolic-parabolic nature can be approximated by an appropriate numerical method based on implicit projection-correction schemes as it is done usually [2, 4]. That involves the resolution of linear systems at every time-stepping of the numerical method.

To circumvent the need to design implicit numerical solvers, we will present an alternative methodology for the numerical approximation based on a hyperbolic approximation of the systems [1, 3]. The most important advantage of this hyperbolic formulation is that it can be easily discretized with explicit and high-order accurate numerical schemes for hyperbolic conservation laws. There is no longer need to solve linear systems as it is usually done in many classical Boussinesq-type systems.

The dispersive features of the first-order formulation and its hyperbolic approximation are studied, and we will show the most relevant results concerning them. We will show a high-order finite volume scheme with the following features: the method is explicit in time, well-balanced for a set of stationary solutions, and can treat wet-dry areas and emerging topographies correctly. Some numerical tests, including idealized academic benchmarks and laboratory experiments, avail the technique's advantage, efficiency, robustness, and accuracy. That constitutes a set of robust and efficient methodologies for coastal type applications.

Acknowledgements

This research has been partially supported by the Spanish Government and FEDER through the coordinated Research Project RTI2018-096064-B-C1 and RTI2018-096064-B-C2, and the Andalusian Government Research Project UMA18-FEDERJA-161.

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[*] Universidad de Córdoba. Email: cescalante@uco.es
[†] Universidad de Córdoba. Email: Tomas.Morales@uco.es