XVIII INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS
THEORY, NUMERICS, APPLICATIONS
Málaga, June 20-24, 2022

Vertical discretization of Euler equations in a shallow water framework

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Shallow water type models have been successfully applied to many real life situations for the simulation of geophysical flows: river floods, sediment transport, tsunami modeling, etc. Nevertheless, shallow water model presents a main drawback: it considers an average horizontal velocity, losing important information about the vertical effects.

In recent years, techniques based on a multilayer approach [1, 3, 5] allows to overcome this simplification and to better describe the flow and vertical effects therein. The idea is to decompose the fluid into M vertical layers, where an average horizontal velocity is considered in each layer.

This not the only possibility to capture vertical effects. For instance, we may consider the shallow water moment model [6, 4]. In this case the variables are considered as polynomials in the vertical direction. This allows again to obtain vertical effects as now the horizontal velocity is not just a constant for the whole column of the fluid.

Another possible approach is to use the so-called σ-coordinate models [2]. In this case a change of variables is performed so that the vertical coordinates varies between 0, the bottom, and 1, the free surface. Then, the free surface is derived from the standard free surface conditions. The interval $[0, 1]$ is then discretized, giving a vertical approximation of the solution of the model.

In this talk we shall review, compare and analyze these different techniques. We propose a general framework that covers them and allows to obtain more complex models that better describe the vertical characteristics of the fluid.

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