XVIII INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS
THEORY, NUMERICS, APPLICATIONS
Málaga, June 20-24, 2022

Sharp critical thresholds in a hyperbolic system with relaxation

Manas Bhatnagar [*], Hailiang Liu [†]

It is a generic phenomena that homogeneous systems of quasilinear hyperbolic systems break down, i.e., the derivative of solutions become unbounded in finite time. The presence of source terms can lead to a delicate balance and persistence of global-in-time solutions for a large set of initial data. The existence of a threshold manifold on the initial phase space so that initial data on one side of the curve results in global-in-time solutions while the other side leads to solutions having shocks/concentration in finite time, is precisely the critical threshold phenomena; see, e.g.,[1].

In [2] we propose and study a one-dimensional 2×2 hyperbolic Eulerian system with local relaxation

$$\rho_t + (\rho u)_x = 0,$$
$$u_t + u u_x = \rho(f(\rho, u) - u),$$

from critical threshold phenomena perspective. The system features dynamic transition between strictly and weakly hyperbolic. For different classes of relaxation we identify intrinsic critical thresholds for initial data that distinguish global regularity and finite time blowup. For relaxation independent of density, we estimate bounds on density in terms of velocity where the system is strictly hyperbolic.

Acknowledgements

This research has been partially supported by the National Science Foundation under Grant DMS1812666.

References

[1] M. Bhatnagar, H. Liu Critical thresholds in one-dimensional damped Euler-Poisson systems. *Math. Mod. Meth. Appl. Sci.*, 30(5):891â916, 2020.

[2] M. Bhatnagar, H. Liu Sharp critical thresholds in a hyperbolic system with relaxation. *Discrete & Continuous Dynamical Systems (DCDS)*, 41(11): 5271–5289, 2021.

[*]Iowa State University. Email: manasb@iastate.edu
[†]Iowa State University. Email: hliu@iastate.edu