XVIII INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS
THEORY, NUMERICS, APPLICATIONS
Málaga, June 20-24, 2022

Spectral stability of shock profiles in hyperbolically regularized systems of conservation laws

Johannes Bärlin *

This talk reports a proof that under natural assumptions on mappings

$$g, f : \mathbb{R}^n \to \mathbb{R}^n,$$

heteroclinic travelling wave solutions

$$u(t,x) = v(x-st), \quad v(\pm\infty) = u^{\pm}$$

to the hyperbolically regularized system of conservation laws

$$g(u)_t + f(u)_x = -u_{tt} + u_{xx}$$

are spectrally stable, if their amplitude $|u^+ - u^-|$ is sufficiently small. This means that an associated Evans function $D : \Lambda \to \mathbb{C}$ with $\Lambda \subset \mathbb{C}$ an open superset of the closed right half plane $\{\lambda \in \mathbb{C} : \mathrm{Re}(\lambda) \geq 0\}$, has only one zero, namely a simple zero at 0.

The result is analogous to one obtained in [1] and [2] for parabolically regularized system of conservation laws,

$$g(u)_t + f(u)_x = u_{xx}$$

and also distinctly extends findings of Mascia and Zumbrun [3] (cf. also Ueda's work with energy estimates in [4]) on the Jin-Xin relaxation case

$$u_t + f(u)_x = -u_{tt} + u_{xx}.$$

It should pave the way towards a treatment of the same question for systems of the form

$$g(u)_t + f(u)_x = B[u]$$

where B is a more general second-order hyperbolic operator, as occur notably in relativistic fluid dynamics [5, 6].

Acknowledgements

This research has been supported by Deutsche Forschungsgemeinschaft under Grant No. FR822/10-1.

References

[1] H. Freistühler and P. Szmolyan. Spectral Stability of Small Shock Waves. *Arch. Ration. Mech. Ana.*, 164: 287–309, 2002.

[2] R. Plaza and K. Zumbrun. An Evans Function Approach to Spectral Stability of Small-Amplitude Shock Profiles. *Discrete Contin. Dyn. Syst.*, 10(4): 885–924, 2004.

[3] C. Mascia and K. Zumbrun. Spectral Stability of Weak Relaxation Shock Profiles. *Comm. Part. Diff. Equ.*, 34: 119–136, 2009.

[4] Y. Ueda. Stability of Travelling Wave Solutions to a Semilinear Hyperbolic System with Relaxation. *Math, Meth. Appl. Sci.*, 32: 419–434, 2009.

*Department of Mathematics and Statistics, Universität Konstanz, 78457 Konstanz, Germany. Email: johannes.baerlin@uni-konstanz.com

[5] H. Freistühler and B. Temple. Causal Dissipation and Shock Profiles in the Relativistic Fluid Dynamics of Pure Radiation. *Proc. R. Soc. Lond. A Math. Phys. Eng. Sci.*, 470, 20140055: 17 pp., 2014.

[6] F. S. Bemfica, M. M. Disconzi and J. Noronha. Causality and Existence of Solutions of Relativistic Viscous Fluid Dynamics with Gravity. *Phys. Rev. D*, 98: 26 pp., 2018.