XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

Transition to oscillatory behavior and breakup of shock profiles in a model of relativistic fluid dynamics

Valentin Pellhammer *

Dissipative relativistic fluid dynamics plays an important role in high energy physics and modern cosmology. Recently new descriptions for it have been proposed. A particularly promising approach for modelling pure radiation fluids has been suggested by Bemfica, Disconzi and Noronha in [1]. This model is given by the four field PDE formulation

$$\partial_\beta(T^{\alpha\beta} + \Delta T^{\alpha\beta}) = 0 \tag{1}$$

with ideal part $T^{\alpha\beta}$ of the energy momentum tensor and dissipative part

$$\Delta T^{\alpha\beta} = -\eta B_0^{\alpha\beta} + \mu B_1^{\alpha\beta} + \nu B_2^{\alpha\beta},$$

where $\eta > 0$ quantifies the viscosity and $\mu, \nu > 0$ are “causalizing“ parameters. This ansatz augments the Eckart description [6] Sec. 2.11, which corresponds to the choice $\mu = \nu = 0$. As it was shown in [1],[3] there is a range $\mathcal{C} \subset \mathbb{R}_+^3$ of parameters (η, μ, ν) that make this model causal.

For an ideal plane shock wave, i.e. a discontinuous solution to the to the Euler equations $T^{\alpha\beta} = 0$ of the form

$$\psi(x) = \begin{cases} \psi_-, & x^\beta \xi_\beta < 0 \\ \psi_+, & x^\beta \xi_\beta > 0, \end{cases}$$

a traveling wave solution $\psi_B(x^\beta \xi_\beta)$ to (1) that has the same end states, $\psi_B(\pm\infty) = \psi_\pm$, is called its dissipative profile and is regarded as the dissipative counterpart of the ideal shock wave.

These profiles thus are heteroclinic orbits of a planar dynamical system of the form

$$B(\psi_B, \eta, \mu, \nu)\psi_B' = F(\psi_B, q) \tag{2}$$

where $q \in \mathbb{R}^2$ is the parameter that identifies the shock wave. In contrast to the situation in [2], the fact that $B(\cdot,\cdot,\cdot,\cdot)$ is in general not definite and not even always regular leads to a rich variety of phenomena as the parameters vary.

While as shown in [3] the mere existence of shock profiles always fails for some shocks for parameter values (η, μ, ν) in the interior of $\mathcal{C}$, the present talk focuses on the situation with (η, μ, ν) on the boundary of $\mathcal{C}$, a case that is referred to as sharply causal since then a characteristic speed of the dissipation tensor equals the speed of light.

We show that in the sharply causal case there are always Lax shocks for which the system (2) has s spiraling focus at ψ_+, so that a corresponding traveling wave oscillates at that end. This oscillatory behavior could well indicate a lack of dynamical stability, as it was observed in other applications [4],[5]. It is associated with a non standard form of saddle-node bifurcation that we discuss in some detail. For still increased amplitude, profiles cease to exist, as the matrix B is singular on a submanifold of the state space.

*Department of Mathematics and Statistics, University of Konstanz, D-78457 Konstanz, Germany, Email: valentin.pellhammer@uni-konstanz.de

References

[1] F. S. Bemfica, M. M. Disconzi, J. Noronha: *Causality and existence of solutions of relativistic viscous fluid dynamics with gravity*, Phys. Rev. D **98** (2018), 104064.

[2] H. Freistühler, B. Temple: *Causal dissipation and shock profiles in the relativistic fluid dynamics of pure radiation*, Proc. R. Soc. A **470** (2014), 20140055.

[3] H. Freistühler: *Nonexistence and existence of shock profiles in the Bemfica-Disconzi-Noronha model*, Phys. Rev. D **103** (2021), 124045.

[4] T. Li, H. Liu, L. Wang: *Oscillatory traveling wave solutions to an attractive chemotaxis system*, J. Differ. Eqs. **261** (2016), 7080 –7098.

[5] R.L.Pego, P.Smereka, M.I.Weinstein: *Oscillatory instability of traveling waves for a KdV-Burgers equation* Physica D, **67** (1993), 45 –65.

[6] S. Weinberg: *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, Wiley, New York, 1972.