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On an inhomogeneous fluid with odd viscosity

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We consider the following system of partial differential equations

$$\begin{aligned} &\partial_t \rho + \nabla\cdot(\rho u) = 0 \\ &\partial_t(\rho u) + \nabla\cdot(\rho u\otimes u) + \nabla\pi + \nu_0 \nabla\cdot(\rho\nabla u^\perp) = 0 \\ &\nabla\cdot u = 0\,. \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^2$ and $t \in \mathbb{R}_+$. In the previous system, ρ, u and p denotes the density, velocity and pressure of the fluid. This system serves as a simplified model of a non-newtonian incompressible fluid with an odd viscosity tensor [1, 2]

$$\mathcal{T}^i_j = -p\delta^i_j + \nu_o\left(\nabla_i u^\perp_j + \nabla^\perp_i u_j\right).$$

In this talk we will present the local well-posedness theory in Sobolev spaces. This result is somehow surprising giving that the previous system is hyperbolic and there is no gain of regularity in u despite the presence of a second order operator. In fact, if one tries to, naively enough, estimate the norm in Sobolev spaces using standard energy estimates, one can conclude

$$\frac{d}{dt}\|\rho\|^2_{H^s} \leq \|\nabla u\|_{L^\infty}\|\rho\|^2_{H^s} + \|u\|_{H^s}\|\rho\|_{H^s}\|\nabla\rho\|_{L^\infty} + lot.$$

Similarly, one can try to estimate the appropriate norm of the velocity field u and find that

$$\frac{d}{dt}\|u\|^2_{H^s} \leq \|\nabla u\|_{L^\infty}\|u\|^2_{H^s} + \|u\|_{H^{s+1}}\|\rho\|_{H^s}\|\nabla u\|_{L^\infty} + lot.$$

To overcome this loss of derivatives, we have to introduce new *good unknowns* that are well-adapted to the structure of the nonlinearities present in the system.

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