XVIII INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS
THEORY, NUMERICS, APPLICATIONS
Málaga, June 20-24, 2022

On the global well-posedness of 3-D density-dependent MHD system

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March 22, 2022

We have studied the problem of existence and uniqueness for the viscous and inhomogeneous magneto-hydrodynamic system which describes the coupling between the density ρ, the velocity u and the magnetic field B equations.

$$(\mathrm{MHD}) \begin{cases} \partial_t \rho + \operatorname{div}(\rho u) = 0 \\ \partial_t(\rho u) + \operatorname{div}(\rho u \otimes u) - 2\operatorname{div}\big(\mu(\rho)\mathcal{M}\big) + \nabla\big(\Pi + \frac{B^2}{2}\big) = \rho f \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad +\operatorname{div}(B \otimes B) \\ \partial_t B + \operatorname{rot}\big(\frac{\operatorname{rot} B}{\sigma(\rho)}\big) = \operatorname{rot}\big(u \times B\big) \\ \operatorname{div} u = \operatorname{div} B = 0 \\ (\rho, u, B)_{|t=0} = (\rho_0, u_0, B_0), \end{cases}$$

Where μ is a positive function denoting the viscosity of the fluid and σ the conductivity. The pressure is denoted by $\Pi(t,x)$ and f represents the volume density of the external forces. The quantity $\mathcal{M}$ is the symmetric part of the gradient: $\mathcal{M} = \frac{1}{2}\big(\nabla u +^t \nabla u\big)$.
We will first recall a number of significant results dealing with existence and uniqueness when the density is constant. As an example, we can cite the work of G. Duvaut and J.-L. Lions [?] in the case of a simply connected bounded domain. These results are completed by M. Sermange and R. Temam [?] for Newtonian fluids. They proved that some classical properties on the Navier-Stokes equations persist for the (MHD) system. In particular the latter is locally well-posed for initial data belonging to the space $H^s,\ s \geq 3$. However the global existence is well known for small data , but unresolved for arbitrarily chosen data.
Before recalling some results for the system (MHD) with variable density, we will set a number of assumptions about viscosity and conductivity. We suppose that σ and μ are of class C^∞ and satisfy:

$$0 < \underline{\sigma} \leq \frac{1}{\sigma} \leq \bar{\sigma} < \infty \quad \text{et} \quad 0 < \underline{\mu} \leq \mu. \tag{1}$$

The existence of global weak solutions in the energy space has been established by J.-F. Gerbeau and C. Le Bris [?] in the case of a simply connected bounded domain. A similar result has been proved by B. Desjardins and C. Le Bris [?] in the case of a torus. The purpose of this article is to deal with the case of strong solutions Fujita-Kato [?], that is, in critical Sobolev-Besov spaces. For this purpose we impose an additional assumption on the initial density ρ_0 of type $\inf_x \rho_0(x) > 0$. Note that this information always remains checked for $\rho(t,x)$ using the maximum principle. We also assume that the density of the fluid is uniform and not zero near infinity, which means that it tends to a finite value at infinity that we can take equals to 1.
The goal is to generalize the work of [?].

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