

Beyond polynomials: SBP operators for general function spaces

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Summation-by-parts (SBP) operators are popular building blocks for systematically developing stable and high-order accurate numerical methods for time-dependent differential equations. The main idea behind existing SBP operators is that the solution is assumed to be well approximated by polynomials up to a certain degree, and the SBP operator should therefore be exact for them. However, polynomials might not provide the best approximation for some problems, and other approximation spaces may be more appropriate. In this talk, a theory for SBP operators based on general function spaces is discussed. We demonstrate that most of the established results for polynomial-based SBP operators carry over to this general class of SBP operators. Our findings imply that the concept of SBP operators can be applied to a significantly larger class of methods than currently known. We exemplify the general theory by considering trigonometric, exponential and radial basis functions.

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