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# Conservation laws and symmetry reductions for a hyperbolic wave equation

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This work is based on a hyperbolic wave equation, with nonlinear damping and source terms, analysed from the point of view of the Lie groups theory. Firstly, we derive low-order conservation laws and discuss their physical meaning by using the multiplier method. Secondly, we apply Lie symmetry method to the hyperbolic partial differential equation to classify the Lie point symmetries. Afterwards, we reduce the partial differential equation to some ordinary differential equations, by using the point symmetries.

The hyperbolic wave equation with damping and source terms is given by

(1) $$u_{tt} - u_{xx} + f(u_t) = g(u), \qquad x \in \Omega,\, t > 0,$$

where $\Omega$ is a bounded domain of $\mathbb{R}^n$, $n \geq 1$.

This equation has been very studied obtaining results concerning non-existence and blow-up solutions [1, 2]. It is well-known that the damping term $f(u_t)$ assures global existence in the absence of the source term ($g(u) = 0$). In fact, the interaction between the damping term and the source term makes the problem more interesting.

The resolution of nonlinear partial differential equations (PDEs) is a very important field of research in applied mathematics. Symmetry reductions have many applications in the context of differential equations. For instance, analytical solutions arising from symmetry methods can be used to study properties such as asymptotic and blow-up behaviour. The symmetries leaving invariant the equation can reduce the number of independent variables, transforming the PDEs into ordinary differential equations (ODEs), generally easier to solve than the original PDE.

Conservation laws analyse which physical properties of a PDE do not change in the course of time. In particular, local conservation laws are continuity equations yielding conserved quantities of physical importance for all solutions of a given equation. For any PDE, a complete classification of conservation laws can be determined by the multiplier method [3, 4].

To sum up, the main goal of this work is to do a complete Lie group classification of equation (1). First, we give a complete classification of the conservation laws admitted by equation (1). Then, we present the reductions obtained from the different symmetries, transforming the PDE into ODEs.

These results are published in [5] in addition to travelling wave solutions obtained by the comparison between equation (1) and similar equations studied previously [6, 7, 8]. However, as a future work, it would be interesting a collaboration to find numerical solutions for the original PDE in order to simulate its behaviour.

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