XVIII INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS
THEORY, NUMERICS, APPLICATIONS
Málaga, June 20-24, 2022

On admissibility criteria for contact discontinuities in Glimm-Isaacson model arising in chemical flooding

Yulia Petrova [*], Dan Marchesin [†], Bradley Plohr [‡]

We will discuss the solutions to a Riemann problem of a non-strictly hyperbolic system of conservation laws ($x \in \mathbb{R}$, $t \in \mathbb{R}_+$):

$$\begin{aligned} s_t + f(s,c)_x &= 0, \\ (cs + a(c))_t + (cf(s,c))_x &= 0. \end{aligned} \tag{1}$$

Here $s = s(x,t)$ is the water phase saturation, $c = c(x,t)$ is the concentration of the chemical agent in the water phase; the function f denotes the fractional flow of water; the function a denotes the chemical's adsorption on the rock. It is commonly assumed that f is an S-shaped function of s for every c, and a is an increasing concave function. This system is often used to describe the displacement of oil by a hydrodynamically active chemical agent (polymer, surfactant, etc) and thus we will call it a *chemical flooding model*.

We will focus on situation with $a \equiv 0$ (zero adsorption). This is the so called Glimm-Isaacson or KKIT model (Keyfiz, Kranzer, Isaacson, Temple, see i.e. [1, 2]). The system has many interesting properties, one of them is the presence of contact discontinuities that causes the non-uniqueness of solutions to a Riemann problem. The standard vanishing viscosity criterion doesn't help, as for contacts there is lack of nonlinear forcing to balance the diffusion terms. There exists an Isaacson-Glimm admissibility criterion, but it lacks the physical motivation. We propose a vanishing adsorption admissibility criterion:

Vanishing adsorption admissibility criterion: a contact discontinuity for the model (1) with zero-adsorption is admissible provided it is the L^1_{loc} limit of a family of admissible solutions of (1) as $\max a(c) \to 0$.

This criterion is physically motivated and we apply it in two situations:

- for the case when fractional flow function f depends monotonically on chemical concentration c, the criterion justifies the Isaacson-Glimm admissibility criterion;
- for the case when fractional flow function f depends non-monotonically on chemical concentration c (that corresponds for some surfactants), the criterion gives rise to the so-called transitional contacts.

The talk is based on the ongoing research with Dan Marchesin and Bradley Plohr, see also a related preprint [3].

References

[1] E. Isaacson. *Global solution of a Riemann problem for a nonstrictly hyperbolic system of conservation laws arising in enhanced oil recovery.* Rockefeller University preprint, 1980.

[2] E. Isaacson, B. Temple. *Analysis of a singular hyperbolic system of conservation laws.* Journal of Differential Equations, 65 (2), pp. 250–268, 1986.

[3] F. Bakharev, A. Enin, Yu. Petrova, N. Rastegaev. *Impact of dissipation ratio on vanishing viscosity solutions of the Riemann problem for chemical flooding model.* arxiv:2111.15001, 2021.

[*]IMPA (Instituto de Matematica Pura e Aplicada), Estrada Dona Castorina, 110, Rio de Janeiro, Brazil, CEP 22460-320. Email: yulia.petrova@impa.br
[†]IMPA (Instituto de Matematica Pura e Aplicada), Estrada Dona Castorina, 110, Rio de Janeiro, Brazil, CEP 22460-320. Email: marchesi@impa.br
[‡]Los Alamos, USA. Email: bradley.j.plohr@gmail.com