XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

Existence of weak solutions and low Mach number limit for quantum Navier-Stokes equations

Paolo ANTONELLI *, Lars Eric HIENTZSCH †Pierangelo MARCATI ‡, Stefano SPIRITO §

The quantum Navier-Stokes (QNS) equations describe a compressible fluid flow subject to a degenerate viscosity stress tensor and a dispersive stress tensor accounting for capillarity effects. The QNS equations can be considered as viscous correction of the Quantum Hydrodynamics (QHD) arising in the description of superfluid flow in Bose-Einstein condensation and superfluid helium. We study the QNS system with non-trivial farfield behaviour providing the suitable framework to study coherent structures and the incompressible limit.

We prove global existence of finite energy weak solutions (FEWS) on the whole space with non-trivial farfield in dimension two and three [3]. Due to degeneracy of the viscosity tensor around vacuum regions no uniform estimates on the velocity field are available. We construct approximate solutions to a truncated formulation of the QNS equations on a sequence of invading domains. Convergence is proven by means of suitable compactness properties stemming from the Bresch-Desjardins entropy estimates. Our result remains valid in the absence of the capillary tensor, namely the compressible Navier-Stokes eq. with density dependent viscosity. This is joint work with P. Antonelli and S. Spirito.

Time permitting, we comment on recent results concerning the low Mach number limit for the 2D and 3D QNS eq. for FEWS [1] and comment on possible generalizations for Navier-Stokes-Korteweg type systems [4]. The main novelty is a precise analysis of the acoustic dispersion [2]. The presence of the dispersive capillary tensor alters the dispersion relation, the linearized system is governed by the Bogoliubov dispersion relation. Based on joint work with P. Antonelli and P. Marcati.

References

[1] P. Antonelli, L.E. Hientzsch and P. Marcati. On the low Mach number limit for the Quantum Navier–Stokes equations, *SIAM J. Math. Anal.* 52 n.6, 6105–6139 (2020).

[2] P. Antonelli, L.E. Hientzsch and P. Marcati. Analysis of acoustic oscillations for a class of hydrodynamic systems describing quantum fluids, arXiv:2011.13435, 2021.

[3] P. Antonelli, L.E. Hientzsch and S. Spirito. Global existence of finite energy weak solutions to the quantum Navier–Stokes equations with non-trivial far-field, *Journal of Differential Equations* 290, 147-177 (2021).

[4] L.E. Hientzsch. On the low Mach number limit for 2D Navier–Stokes–Korteweg systems, to appear in *AIMS Press Mathematics in Engineering, Special Issue: Fluid instabilities, waves and non-equilibrium dynamics of interacting particles* (2022).

*Gran Sasso Science Institute, viale Francesco Crispi, 7, 67100 L'Aquila, Italy. Email: paolo.antonelli@gssi.it
†Fakultät fr Mathematik, Universität Bielefeld, Universitätsstrasse 25, 33615 Bielefeld, Germany. Email: lhientzsch@math.uni-bielefeld.de
‡Gran Sasso Science Institute, viale Francesco Crispi, 7, 67100 L'Aquila, Italy. Email: pierangelo.marcati@gssi.it
§DISIM - Dipartimento di Ingegneria e Science dell'Informazione e Matematica, Via Vetoio, 67100, L'Aquila, Italy. Email: stefano.spirito@univaq.it