Compactness Property of the Linearized Boltzmann Operator for Polyatomic Gases

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We consider the Boltzmann equation that models a polyatomic gas by taking into account the continuous microscopic internal energy I. In particular, we consider the kinetic system proposed by [2], which is based on the procedure of Borgnakke and Larsen [1]. We linearize the Boltzmann equation around the Maxwellian function, which represents the equilibrium distribution function. Under some convenient assumptions on the collision cross-section $\mathcal{B}$, we prove that the linearized Boltzmann operator $\mathcal{L}$ is a Fredholm operator. For this, we write $\mathcal{L}$ as $\mathcal{L} = \mathcal{K} - \nu.I$, and we prove that $\mathcal{K}$ is a compact operator. The compactness is achieved as a result of $\mathcal{K}$ being a Hilbert-Schmidt integral operator. This work was indeed done by Grad [3] for a monoatomic single gas, and by Pavic [4] for a mixture of monoatomic gases.

References

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