

Construction of solutions of a two-dimensional Riemann problem for a thin film model of a perfectly soluble anti-surfactant solution

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This article is concerned with formulation of three-dimensional thin film model for an anti-surfactant solution and hence constructing unique global solution for a two-dimensional Riemann problem for the corresponding reduced hyperbolic form obtained by considering that the solute is perfectly soluble and the capillarity and diffusion effects are negligible. We develop six geometrically different structures of the solution of this Riemann problem using generalized characteristic analysis method while relaxing the restriction that only one planar elementary wave is developed at the interface of each initial discontinuity. It is interesting to observe that the hyperbolic system under consideration belongs to an important class, Keyfitz-Kranzer class, of hyperbolic system of conservation laws and therefore, for certain cases of initial data we consider non-classical delta shock waves as well. Also, we analyze the interactions of classical and non-classical waves in detail to construct the global solution of the corresponding 2-D Riemann problem. Further, we provide the expressions for strength, location and propagation speed of delta shock wave at each interaction point. Moreover, we compare these solutions with the solutions of a one-dimensional rotated initial value problem and prove that our solutions are globally unique.

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